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VISUAL STUDIES OF RADIO FADING.[1]

BY

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In recent years there has been an increasing tendency to include under the general term "fading" not only the variation in loudness of a radio signal but also changes in tone quality, in polarization, in phase—in fact changes in any of the characteristics of the radio waves by which the signal is transmitted. It is in this broad sense that the term is used in this article. In a few cases certain types of fading may perhaps be explained as the result of irregularities in the behavior of the transmitter. But in general the phenomena are undoubtedly caused by changes in the atmospheric conditions that prevail along the path, or paths, by which the electric waves proceed from the transmitter to the receiver. The study of fading phenomena thus has a double interest, since it is one of the few methods available for determining conditions in the atmosphere at points that cannot be reached by direct observation.

The waves radiated from a broadcasting station ordinarily proceed to the receiving station by at least two different routes. The "ground wave" follows the surface of the earth

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—approximately, at least, along the arc of a great circle. The "sky wave" proceeds obliquely upward from the sending station and reaches the observer after reflection from the Kennelly-Heaviside layer. During the day-time the sky wave is usually weak and transmission is almost entirely by means of the ground wave. But at great distances from the sending station the ground wave becomes so weak that it is a negligible factor and transmission is by means of the sky wave. In the case of short waves this condition exists at all distances except in the immediate neighborhood of the sending station. Under conditions which are not yet very well understood the sky wave may suffer repeated reflections back and forth between the Heaviside layer and the earth. And under other conditions, which are still less clearly understood, the signal seems to be carried by several different sky waves proceeding by different routes, but without multiple reflection, and uniting at the receiving station.

Changes in the ionization of the atmosphere between the sending and receiving stations may affect the radio waves in three ways: (1) the amplitude may be changed because of absorption; (2) changes in the index of refraction of the air will alter the shape and effective length of the path followed by the waves; and this in turn will change the phase of the waves that reach the receiving station by the path in question; (3) because of the magnetic field of the earth there will be a rotation of the plane of polarization of each beam and a separation of the two polarized components by magnetic double refraction. Since the response of the receiving apparatus is determined by the resultant field at the receiving station, and since the different wave trains which contribute to this resultant follow different paths and are probably influenced by atmospheric conditions to a different extent, it is clear that the phenomena of fading are likely to be very complicated.

In order to utilize observations of fading to the best advantage as a means of studying the influence of atmospheric conditions on radio transmission we should be able to observe and correlate as many as possible of the quantities which serve to describe the resultant field at the receiver. The records of fading phenomena that have been most frequently made show the variation from instant to instant in the intensity of the signal received by a vertical antenna. Such records give information regarding variations in the amplitude of the vertical component of the resultant electric field. By means of coil receivers—which, however, have not been very extensively used—it is possible to determine the variation in amplitude of the different components of the magnetic vector. But to describe the phenomena completely we must know the variation from instant to instant in the amplitude and phase of each of the components of the electric and magnetic vectors at the receiving antenna.

A method has been described by two of the authors [2] by which it is possible to determine conveniently both the amplitudes and the relative phase of two components of the resultant magnetic field at the receiver. The method may be readily adapted to the comparison of any two of the quantities involved. It is therefore possible in principle to make observations which give a complete description of the phenomena. While nothing so elaborate as this has been attempted in the work described in this paper, the method has been used extensively as a means of comparing two components of the field at a given point, or at neighboring points, in the attempt to gain a clearer understanding of the causes of fading and the relative importance of the known causes at different times.

The observations reported in this paper were made with the signals from commercial broadcasting stations. Similar observations with short waves are in progress.

APPARATUS AND OBSERVATION TECHNIQUE.

The general plan of observations was to place the carrier wave (or an equivalent audible heterodyne which preserves the variations of amplitude and phase of the original signal) picked up by two antennæ, upon the deflector plates of a cathode ray oscilloscope tube, and by observing the changing figures drawn on the fluorescent screen by the rapidly moving electron beam, to study the differential variations in amplitude and phase of the signals received at the two antennæ.[3]

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[2] Merritt, E., and Bostwick, W., *Proc. National Academy of Sciences*, 14, p. 884, 1928.

The requirements placed on the receiving system were as follows:

1. Sufficient sensitivity must be provided to receive signals from broadcasting stations within a reasonable distance under average daytime conditions, using directive loop antennæ not too large to be conveniently portable.
2. The receivers must have a selectivity good enough to separate the carriers of stations on adjacent channels when the signals being separated are of about equal intensity.
3. Shielding and antenna coupling must be so designed that the only signal indicated by the receivers is that actually received on the loop antenna, unmixed with anything picked up anywhere else in the system.
4. The receivers must be supplied in duplicate, and operated with a common heterodyne; the method of introducing the common heterodyne must not introduce appreciable coupling between the individual receivers.
5. The loop antennæ must be readily portable over a flat open field for distances up to a thousand feet from the receivers.
6. The loop antennæ must give true direction readings, and have no appreciable linear antenna pick-up.
7. The feed-lines must be rugged and easily handled; of reasonable weight and cost, and easily portable.

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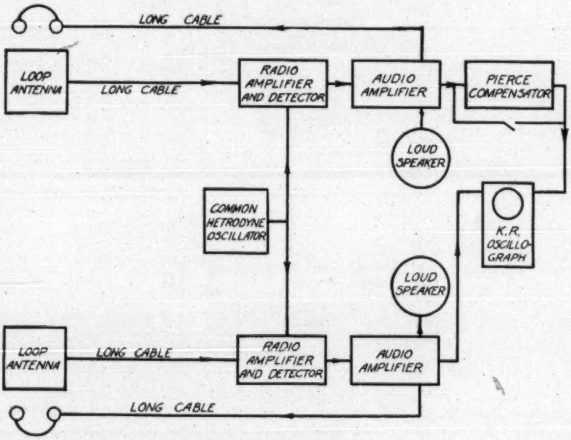
[3] Merritt and Bostwick, l.c. The method of determining the relative amplitudes and phases of two waves by means of the corresponding beat tones from a common heterodyne was developed by one of the writers in 1918 at the Naval Experimental Station, New London, Conn., the main purpose then being to make possible an adaptation of the "binaural" methods of direction finding to radio signals. With other devices developed during the war by members of the scientific staff it was later patented. (U. S. Patent No. 1,510,792. "Methods and Means for Determining Phase Difference" issued to Ernest Merritt.) The procedure used by the authors to make the method visual had been used by H. T. Friis with short wave signals received on vertical antennæ spaced about one third wave-length apart and has led to an interesting modification of the binaural method in which phase differences are detected by the eye instead of the ear. (H. T. Friis, "Direction of Propagation and Fading of Short Waves," *Proc. I. R. E.*, **16**, p. 658, 1928.) The combination of these two methods as used in the experiments herein described was developed by Merritt and Bostwick (E. Merritt and W. E. Bostwick, *Proc. National Academy of Sciences*, **14**, p. 884, 1928) and expanded to the present form by the authors.

8. The audible frequency amplifiers must not introduce either appreciable frequency discrimination or phase distortion.

9. To secure sufficient amplitude in the cathode ray oscilloscope the audio amplifiers must supply an output of 15 volts maximum into an extremely high impedance load; and must have as much amplification as is usable. This means that the audio amplifier must have a maximum amplification great enough to make the detector tube and circuit noise just visible on the oscilloscope screen.

10. The total amplification of the system must be adjustable over a considerable range, and when once set must not change more than a few per cent during a period of several hours.

FIG. 1.


LONG CABLE
LOOP ANTENNA
LONG CABLE
RADIO AMPLIFIER AND DETECTOR
AUDIO AMPLIFIER
PIERCE COMPENSATOR
LOUD SPEAKER
COMMON HETRODYNE OSCILLATOR
K.R. OSCILLOGRAPH
LOUD SPEAKER
LOOP ANTENNA
LONG CABLE
RADIO AMPLIFIER AND DETECTOR
AUDIO AMPLIFIER
LONG CABLE

Schematic general layout.

The general arrangement of apparatus is shown in the block diagram (Fig. 1). The antennæ, radio receivers, and amplifiers are duplicated, supplied with a common heterodyne, and worked into an oscilloscope, the two complete channels each operating one pair of deflector plates. For some of the work a Pierce electric compensator was inserted in one channel after the audio amplifier to alter the phase of one channel relative to the other by a measured amount.

A considerable variety of types, locations, and directions of antennæ were used in various combinations, but most of the work was done with balanced frame antennæ of the kind frequently used with direction finding receivers. The detailed circuits of two types of antenna connections are shown in Fig. 2. Diagram *a* is the original circuit, while *b* shows the most recent improvements. In Fig. 2*b* the feed line was conductively coupled at both ends, the mutual inductances being adjusted to match the surge impedance of the cable when the networks were tuned to resonance. Only the voltage induced in the loop is allowed to reach the amplifier. Linear antenna pickup of the unshielded cable and loop

FIG. 2*a*.

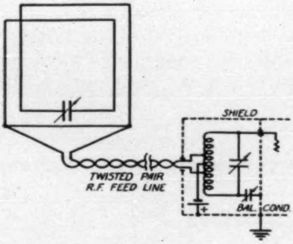
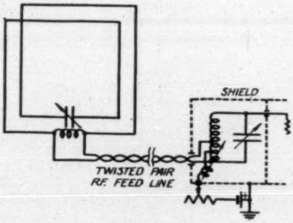


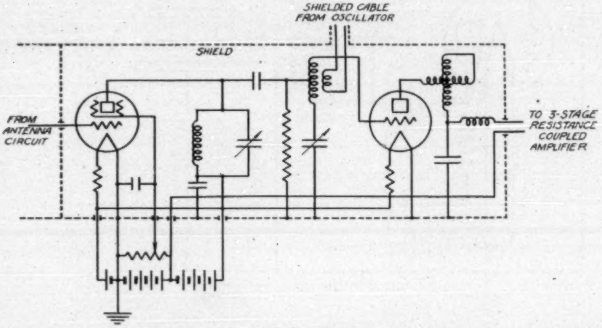
FIG. 2*b*.



as a whole is carefully balanced out at the amplifier input transformer, by means of a fine adjustment of the ratio of inductance to ground from each line wire, and an absorbing resistance.

When it was desired to shift the observations rapidly from one station to another on a different frequency, aperiodic loop antennæ were used to save time. The aperiodic loops consisted of a single turn of wire on a light frame about 1 meter square. The ends of the turn were connected directly to the cable. With signals of average to heavy field strengths, good results were obtained, but with weak fields it was necessary to use tuned loops. Comparative tests indicated that the transmission loss in a 250 meter cable including the transforming networks at both ends was about 2.5 db as compared to the same tuned loop worked directly into the first amplifier. This is for the circuit of Fig. 2*b*. The loss for 2*a* was about 1 db greater. The loss for the aperiodic loop as compared with a tuned loop of nearly the same size was about 20 db.

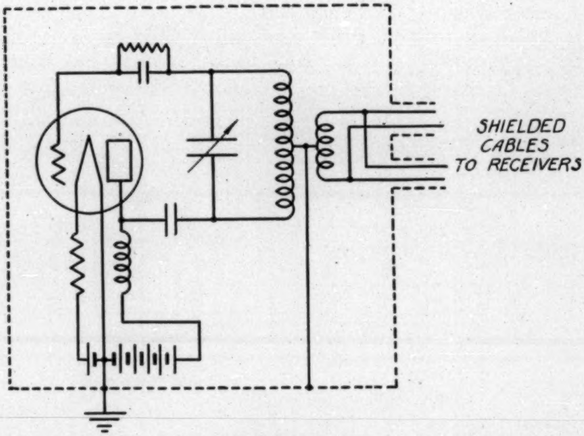
FIG. 3.


SHIELDED CABLE FROM OSCILLATOR
SHIELD
FROM ANTENNA CIRCUIT
TO 3-STAGE RESISTANCE COUPLED AMPLIFIER

Receiver circuit.

A circuit diagram for one of the receivers is shown in Fig. 3. The receivers have several interesting features: (1) the screen-grid radio-frequency amplifier stage completely blocks off any feed from the common heterodyne or oscillating detector to the antenna circuits; (2) the amplification of this stage is much higher than the values ordinarily realized in commercial receivers because the regenerative detector raises the resonant impedance of the plate circuit load of the screen-grid tube to almost any desired value.[4] The amplification of the stage therefore approaches the amplification factor of the tube, in this case from 300 to 400 voltage ratio, most of which is actually realized, because the application of regeneration to the input transformers has been found to result in small gain due to the fact that only the loss of the input transformer is thereby eliminated, and this is only a moderate portion of the entire transmission loss of the antenna system. Simple tuned impedance coupling was used because it gives the largest amplification. A regenerative detector circuit of the tuned grid and tuned plate type was used because of its high sensitivity and flexibility of control,

FIG. 4.



Oscillator circuit.

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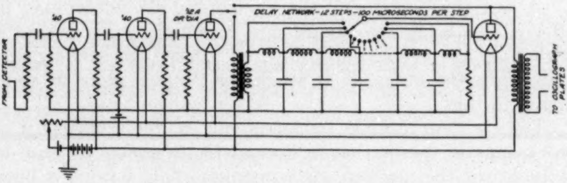
[4] See "Shielded-grid Pliotrons," by A. W. Hull and N. H. Williams, *Physical Review*, April, 1926.

offering adjustable selectivity. A third interesting feature is the manner in which the separate heterodyne is introduced. A balanced transformer in a bridge type circuit introduces the heterodyne voltage from a shielded feed line to the detector grid, without introducing any coupling between the two receivers through the common heterodyne oscillator.

The common heterodyne is supplied by a simple Hartley oscillator shown in Fig. 4. The only thing to note here is that the output is fed to the two receivers through two ordinary BX cables connected in parallel, the circuits being balanced by a ground at the center of the coupling coil. Voltages of the same magnitude and phase in the two detector circuits were insured by making the input circuits of identical measured impedance, and cutting the cables to the same length.

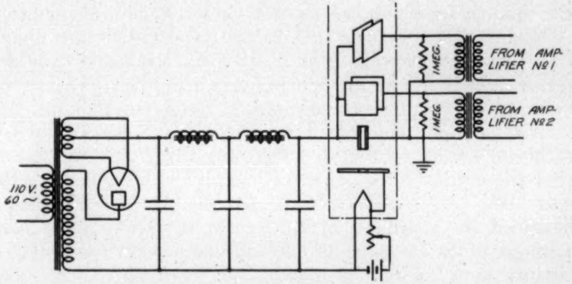
FIG. 5.



Amplifier and compensator circuit.

Fig. 5 details the audible amplifiers and delay circuit. The entire delay circuit was cut out for most of the observations. Care was taken to adjust the voltages and circuit constants of the audible amplifier circuits to keep phase distortion as small as possible. Tube overloading could be detected at once on the oscilloscope. When using the delay circuit the heterodyne frequency was adjusted to give a beat tone tuned to a standard tuning fork so that the phase angle of each step of the compensator could be computed. Calibration of the delay circuit was checked against computed values by means of an artificial signal. Tuning forks and steel bars were then adjusted to whatever pitch was necessary to give the delay circuit an exactly prescribed phase angle per step. The oscilloscope circuit is shown in Fig. 6.

FIG. 6.



Oscillograph circuit.

OBSERVATIONS WITH CROSSED LOOPS.

A partial separation of the ground and sky waves may be effected by using two loops, one of which, A, is mounted with its plane vertical and directed toward the sending station, while the other, B, is set in the vertical plane at right angles to this.

Since a loop receiver responds only to the component of the magnetic vector that is normal to its plane, a signal in A indicates the presence of a magnetic field having a horizontal component at right angles to the line to the sending station, while the signal in B is due to a magnetic vector having a component directed toward, or away from, the station. It will simplify the discussion to make use of a system of rectangular coördinates having its origin at the loop. Let OX be directed toward the sending station, OY the vertical, and OZ a horizontal line at right angles to OX. Then loop A responds to H_z and loop B to H_x, these being the Z and X components of the resultant magnetic field at the loop.

In the case of short waves there is some evidence that at a great distance from the sending station the line of propagation of the waves may sometimes deviate slightly from the great circle connecting the two stations. So far as we are aware, however, there is no indication of any such deviation in the case of the waves used in ordinary broadcasting, i.e. in the wave-length range from 200 m. to 600 m., and there are many reasons for believing that if there is such a deviation, it is quite small. We shall assume therefore that the line of propagation of the waves lies in the plane *XOY*.

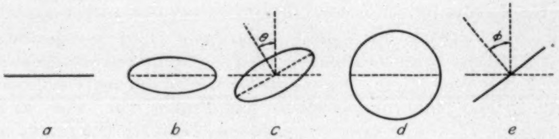
In general the field at *O* will be the resultant of the fields due to three separate wave trains; i.e. the "ground wave" proceeding in the direction *OX*; the "sky wave" coming obliquely downward; and the reflected sky wave proceeding obliquely upward. The ground wave has its electric vector in the plane of incidence and vertical except for a slight forward tilt which may ordinarily be neglected. Its magnetic vector is therefore in the direction *OZ* and can produce no effect in the loop *B*. The sky wave may have its electric vector in any direction normal to the direction of propagation and in general is elliptically polarized. This is equivalent to regarding it as consisting of two polarized components which differ in phase, the magnetic vector of one component lying in the plane of incidence, and that of the other in the direction *OZ*. The latter component, both for the direct and reflected beam, will produce an effect in *A* which is superposed on the effect due to the ground wave. On the other hand the component of the sky wave which has its magnetic vector in the plane of incidence will produce an effect in *B* only.

It will be noticed that no signal will be received in *B* unless a sky wave is present; and in the case of waves received from a station not more than one or two hundred miles distant, the effect of the ground wave in loop *A* is usually considerably greater than that of the sky wave. When the two loops are connected to the oscillograph so that *A* produces a horizontal movement of the spot while *B* produces a vertical movement we thus obtain a partial separation of the ground and sky waves which is always helpful and in many cases quite sufficient.

In general the oscillograph figure was an ellipse, which might become a horizontal straight line if the amplitude of the signal in *B* was zero, a vertical straight line if the amplitude in *A* was zero, or an oblique straight line if the two signals were in phase. Before each observation the two receiving channels were adjusted to give the same amplification and phase shift. This was done by placing the two loops parallel and near together (but not coupled). The desired station was then tuned in, the amplification adjusted for equal amplitudes on the oscilloscope axes, and the phase shifts adjusted to equality by fine tuning of the resonant networks until the figure on the screen was a straight line with an inclination of 45° from the axes.

The oscilloscope figure gives a graphical indication of the conditions that lead to the "night errors" of a radio compass, which is often instructive. If the figure is a horizontal line (Fig. 7*a*) the resultant magnetic vector is at right angles to the line to the station. In this case a vertical coil used as a direction finder will give the true bearing of the sending station. If the figure is elliptical (*b* and *c*, Fig. 7) no position can be found for which the signal in a vertical coil is zero, and the radio compass must be set to a minimum. For the case represented in Fig. 7*b*, the bearing will nevertheless be correct; but if conditions corresponding to Fig. 7*c* prevail the apparent direction of the station will be the direction of the minor axis of the ellipse and the bearing will be incorrect by the angle θ. If the ellipse becomes a circle (Fig. 7*d*) the signal will be the same for all settings of a vertical loop and the radio compass is useless. When the ground and sky waves are in phase, so that the oscilloscope figure takes the form of a tilted line (Fig. 7*e*) we have the condition that is the most dangerous source of error; for while the radio compass will set to a sharp zero, the apparent direction will be at right angles to the resultant field and therefore in error by the angle φ.

FIG. 7.



On one occasion when the conditions were unusually steady, signals from station WHAM (Rochester, N. Y., about 60 miles N.W. of Ithaca) gave an oscilloscope figure of the type shown in Fig. 7*e*, which remained unchanged from about 8 P.M. until 9 P.M., when we shifted to another station. During this whole time a third loop, used as a direction finder, gave a sharp zero when set in the plane of the meridian. If we had not known that the true direction of the station was not west but northwest, the sharpness and constancy of the settings, and the complete absence of fading, would probably have led us to accept the indications of the radio compass with full confidence. Figures corresponding to all of the typical cases of Fig. 7 have been repeatedly observed.

Observations with crossed loops were begun during the summer of 1928 and were continued, although with frequent interruptions, during the summers of 1929 and 1930. The apparatus was located at the base of a wooden tower near the center of Alumni Field, a level field about 750 ft. long in the East and West direction and about 2000 ft. wide (in N.S. direction). The A.C. current supply was brought to the tower in an underground cable.

During the day time the oscilloscope figure is usually a horizontal straight line of constant, or very slowly changing, length. Day time transmission from broadcast stations is effected apparently chiefly by means of the ground wave—or, if a sky wave is present, its electric vector lies almost exactly in the plane of incidence.[5] Exceptions to this general rule are, however, not unusual. Almost all stations occasionally show an appreciable sky wave during the day time. And one station (WGY, Schenectady) has rarely been without a sky wave, even in the middle of the day, the vertical amplitude of the oscilloscope figure being usually from 10 per cent. to 20 per cent. of the horizontal amplitude.

As sunset approaches indications of a sky wave begin to appear. The horizontal line slowly pulsates, or tilts back and forth about the horizontal. In other cases it opens up into a narrow ellipse which changes in area and inclination. (See Fig. 8.) The time at which these changes begin varies greatly from day to day. Sometimes the figure shows large and rapid changes two or three hours before sunset, while on other occasions daylight conditions persist for an hour or more after darkness has set in.[6]

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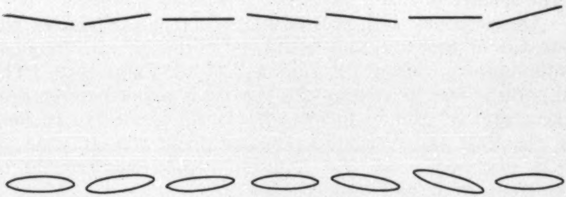
[5] It must not be forgotten that a sky wave having its electric vector in the plane of incidence would not be detected by the method used but would appear as a part of the ground wave. Polarization with the electric vector in the vertical plane is to be expected in the case of most of the antennas used for broadcasting. Sky waves from such antennas would therefore produce an effect in loop *B* only in case they have gone high enough for the plane of polarization to have been appreciably rotated by the Faraday effect in the earth's magnetic field. The presence of a sky wave which reaches the receiving station with an angle of incidence not far from 90 degrees seems not improbable.

FIG. 8.



When night conditions have been established—usually about an hour after sunset—the movements of the oscilloscope figure are in general very erratic. The figure often gives the appearance of rotation in the plane of the screen, although the rotation is ordinarily not continuous but back and forth through a varying angle—usually less than 180 degrees—and with varying rapidity. At the same time there is usually a continuous change in the vertical and horizontal amplitudes. The two rarely change in synchronism. The vertical amplitude may go to zero while the horizontal amplitude is large, or vice versa. Added to this we have phase changes, which still further alter the shape and orientation of the figure.

Since the changes in the oscilloscope figure were usually too rapid to be recorded photographically, and too erratic to be described by words, our method of recording them was to make rough sketches, at intervals of a few seconds, to show the variation in the shape and size of the figure as it passed through a sequence of changes. Several such sequences, reproduced from our note books, are shown in Fig. 9,

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[6] A striking instance of this variation from day to day in the time at which the transition occurs between day and night conditions was shown by radio compass observations made in connection with the solar eclipse of Jan. 24, 1925. See E. Merritt, C. C. Bidwell, and H. J. Reich, JOURNAL OF THE FRANKLIN INSTITUTE, April, 1925.

FIG. 9*a*.

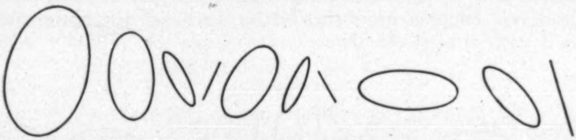


FIG. 9*b*.

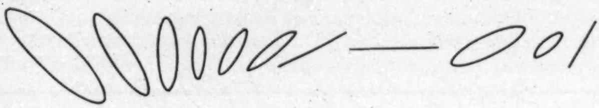


FIG. 9*c*.

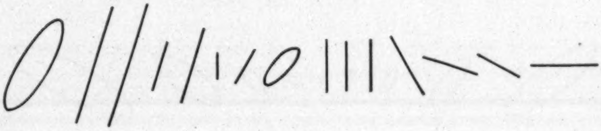


FIG. 9*d*.

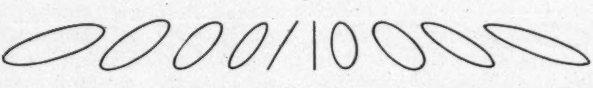
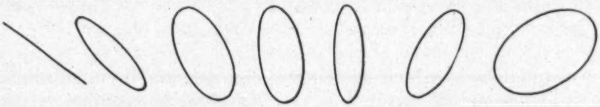


FIG. 9*e*.



a to *e*. Figure 9*a* is typical of conditions where the changes occurred with no suggestion of system. In the sequence shown in Fig. 9*b* there is clockwise rotation combined with phase changes and amplitude fading, and changing toward the end to rotation in the opposite direction. This was a very common type of sequence. In Fig. 9*c* the horizontal and vertical vibrations (ground and sky waves) remained in the same phase during the greater part of the time, but show amplitude variations which seem to be quite independent of one another. Figures 9*d* and 9*e* begin to show evidence of systematic changes involving both rotation of the figure combined with changes in size.

Fig. 10*a*.

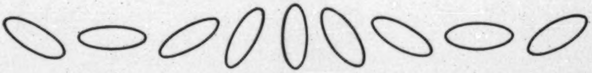


Fig. 10*b*.

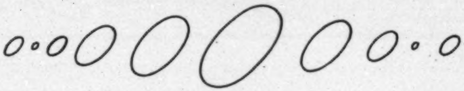


Fig. 10*c*.

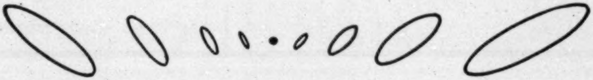


Fig. 10*d*.

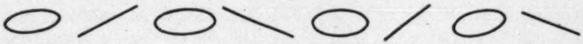
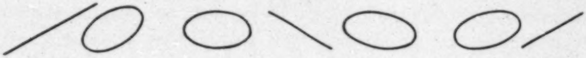


Fig. 10*e*.



Sequences in which systematic changes are very definitely in evidence are shown in Fig. 10. Of these the sequence 10*a*, showing continuous rotation, and 10*b*, showing periodic change in size without change in shape or orientation, are the most striking. They were also more frequently observed than any of the other systematic sequences, although rarely in as perfect form as shown here.

In spite of the extremely irregular and erratic movements of the oscilloscope figure it is possible to make a few general statements which apply to all of our crossed loop experiments:

1. Except possibly during the sunset transition period, amplitude fading, if present at all, is usually present in both the horizontal and vertical amplitudes. If big changes are observed in one case they are likely to be observed also in the other. And if the changes are particularly rapid (or slow) in the horizontal amplitude they are also rapid (or slow) in the other.

2. When there is a change in the general character of the fading phenomena—e.g. if rapid fading changes to slow fading, or if conditions causing large amplitude changes pass into conditions where only small fluctuations of amplitude occur—the change occurs gradually. Whatever the conditions are that determine the general character of the fading phenomena they change progressively, not discontinuously.

3. If the oscilloscope figure for one station is unusually active in its movements the figures for other stations are likely also to show unusual activity. Or if the conditions are quiet for one station they are likely to be so for all. It is not unusual, however, for the stations to be grouped according to direction in their fading behavior. On several occasions, for example, we have found the oscilloscope figure for stations lying west of Ithaca nearly constant in size and shape at the same time that stations to the east gave rapidly changing figures.

4. When the oscilloscope figure is changing in shape and orientation, as well as in size, the changes in shape and orientation are often relatively slow when the figure is large and become much more rapid when the amplitudes are small.

5. If for one station the vertical amplitude ("sky wave") is larger (or smaller) than usual this is likely to be true for all stations. Here too, however, there is sometimes a grouping according to direction.

We shall refer later to certain difficulties that stand in the way of a complete explanation of the crossed loop observations on the basis of the generally accepted theories. But in a general way, and considered from the qualitative standpoint only, the results are exactly what might be expected.

The absence of a sky wave during the day time is explained either by the increased absorption due to high ionization or to the fact that the Kennelly-Heaviside layer is so low that the plane of polarization is not appreciably rotated. The gradual appearance of a sky wave near sunset is due to the rise in the reflecting layer, and to the decrease in atmospheric absorption that result from the decreased ionizing power of the solar radiation. The time at which the transition to night conditions begins, and the manner in which it takes place, will be determined by the conditions in the upper atmosphere—e.g. pressure, speed and direction of motion, constitution (dust, moisture). After night conditions are established, changes in phase between the ground and sky waves are to be expected because the two follow different paths and along each of the two paths the conditions are continually changing. Amplitude fading, especially if slow, may be thought of as due to changes in the conditions that cause absorption. Rapid changes in amplitude may be accounted for as the result of interference between the ground and sky waves, sometimes additive and sometimes subtractive. This accounts very nicely, for example, for the pulsating ground wave that is often observed as sunset approaches. Rotation of the plane of polarization will often account in a very satisfactory way also for many of the observed amplitude changes. Since all the phenomena of fading are due to conditions in the atmosphere, the fact that all stations—or groups of stations in a given direction—tend to show similar behavior (Paragraphs 2, 3 and 5) is merely an indication that the conditions in the upper atmosphere are on any given occasion much the same over the whole country, or a large part of it.

When these suggested explanations of the phenomena are considered more in detail, however, they are found in many cases to be inadequate or to involve quantitative assumptions that are not admissible. Consider, for example, the behavior of the sky and ground waves as regards amplitude fading. In the vast majority of cases big or rapid fading of the horizontal amplitude of the oscilloscope figure is accompanied by big or rapid fading of the vertical amplitude (Paragraph 1 above). Now changes in the amplitude of the horizontal vibration are readily accounted for by interference between the ground wave and that component of the sky wave whose electric vector is vertical. But if there is only a single sky wave changes in the vertical amplitude cannot be explained by interference; for the vertical vibration is caused by the component of the sky wave whose magnetic vector is in the line toward the station, and the ground wave has no component that can interfere with this. Amplitude fading might also be caused by a rotation of the plane of polarization of the sky wave and if due to this cause—combined probably with interference effects between the ground and sky waves—the fading would be about equally great for the horizontal and vertical amplitudes. But the two would be out of step—maximum vertical amplitude would occur when the horizontal amplitude is least, and vice versa. Cases when this is true do occur, but they are the exception.

Although the movements of the oscilloscope figure were ordinarily quite erratic there were times when there was unmistakable evidence of system. For example, on a number of occasions, about fifteen minutes after sunset, the figure was observed to rotate continuously in the same sense, sometimes changing at the same time from an ellipse to a line and back again. (See Figs. 10*a* and 9*d*.) No correlation was noticed between the rotation and the change of shape. The period of rotation was different on different occasions but was usually not less than 5 sec. nor more than 20 sec. As many as twenty complete turns have sometimes been counted in a period of a few minutes. Continuous rotation of this kind was sometimes observed at night also but usually did not last as long as at sunset. At night a period of rotation in one sense was likely to be followed a little later by rotation in the opposite sense. At sunset no reversal of this sort has been noticed. In cases of rotation the length of the major axis of the figure remained nearly constant.

There can be little doubt that we are dealing here with the same phenomenon that was detected by Parkinson[7] by simultaneous observations of the signal intensity from two loops directed respectively toward the station and at right angles to this direction, when it was found that for considerable periods the minimum signal intensity in one loop occurred at the same time as the maximum in the other.[8] The period, from 3 sec. to 30 sec., reported by Parkinson for this type of fading, agrees with that observed by us; and the time of occurrence (at sunset for Station WBAL and from 15 to 30 minutes later for Station WJZ) is that at which we found a rotating figure most common.

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[7] Parkinson, T., "Some Observations of Short Period Radio Fading," *Proc. Inst. Radio Engineers,* 17, p. 1042, 1929.

The phenomena seem to be best explained by rotation of the plane of polarization due to the earth's magnetic field. If this explanation is adopted it is necessary, however, to assume very rapid changes in the length of the path followed by the sky wave. If the rotation occurs at the upper part of the path, where the electron density is presumably of the order 10^5, one complete rotation [9] will be produced in case λ equals 400 m. in about 2000 m. In cases, where as many as 20 complete turns have been observed, we must therefore assume that the effective path length changes by as much as 40 km. in a few minutes. The time when these cases of continuous rotation were observed—shortly after sunset—is, however, exactly the time at which such rapid changes in path length are to be expected.

A type of systematic fading that was of more frequent occurrence was characterized by changes in the oscilloscope figure of such a nature that the vertical and horizontal amplitudes changed in the same proportion, so that the figure, while changing in size, often quite rapidly, nevertheless, retained its shape. Cases where fading of approximately this type was observed were quite frequent. A case of almost exact constancy in shape is shown in Fig. 10*b*.

It is difficult to see how fading of this sort can be explained on the assumption of a single sky wave. Interference between the sky and ground waves might cause the observed changes in the horizontal amplitude but could not affect the vertical amplitude. Changes in the absorption by the atmosphere, even if sufficiently rapid, could not account for the observed effects unless the region of changing absorption was in the immediate neighborhood of either the sending or receiving station, so that the sky and ground waves would be equally affected. This seems extremely improbable. The frequent occurrence of this type of fading is one of the strongest of the reasons, of which there are many, for believing that sky waves reach the receiving station by several different paths and that interference between these different sky waves is an important factor in determining the observed fading.

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[8] With short waves the same type of effect is indicated by the fact that simultaneous fading records taken with a vertical and horizontal antenna are opposite in phase. Eckersley, T. L., "Polarization and Fading of Short Wireless Waves," *Nature*, **121**, p. 707, 1928. Our own observations on short waves by the visual crossed loop method (not yet published) show that a rotating figure is very common.

[9] See Peddersen, "The Propagation of Radio Waves," p. 208, Copenhagen, 1927. The calculation is there made for ω equals 10^8, i.e. $\lambda = 36$ m. The value here given is on the assumption that the distance corresponding to one rotation varies as ω^2.

DIFFERENTIAL FADING BETWEEN STATIONS LESS THAN A WAVE-LENGTH APART.

Simultaneous observations of signal strength at different stations have shown that even at stations only a few miles apart the fading may be quite different. In general such observations have shown that slow changes in intensity are likely to occur in approximate synchronism over a large area, but that with rapid fading synchronism is much less in evidence and may be entirely absent. It is sometimes found that the fading is quite different even at stations only a few miles apart.

The visual method that we have employed lends itself especially well to the study of differential fading and has the advantage of indicating differences of phase as well as differences in amplitude.

To compare the fading phenomena at two stations *A* and *B* a loop antenna was placed at each station and the signals were brought to the two receiving sets by cables. The two loops were in all cases parallel to one another, and were directed either toward the sending station or in the plane at right angles to this direction.

The receiving sets and the mode of connection to the oscilloscope were the same as previously described. Thus the horizontal vibration on the oscilloscope was due to the signal received at *A* while the vertical vibration was produced by the signal arriving at *B*. By adjusting the amplification and by slight changes in the setting of the first tuning condenser the two vibrations were adjusted both to the same amplitude and to the same phase. Any changes in amplitude or phase that occurred at *A* and *B* after this adjustment was made were then shown by a change in the oscilloscope figure. Thus if the fading was in every respect the same at the two stations the figure would simply increase or diminish in length, remaining, however, a straight line inclined at an angle of 45 degrees to the horizontal. If the signals at *A* and *B* remained in the same phase but changed in relative amplitude the line tilted back and forth about the 45 degree position. If a change occurred in the relative phase at the two stations the line opened out into an ellipse.

On one occasion observations were made with the stations a whole wave-length apart. Usually, however, the distance between *A* and *B* was from $\lambda/4$ to $\lambda/2$. The loops *A* and *B* were so placed that the line joining the two either pointed toward the sending station or was at right angles to this direction.

In the case of short waves previous observations have sometimes indicated a deviation from the great circle path. But we do not know of any results which definitely suggest an appreciable deviation from this direct path with waves in the broadcast band (200 m. to 600 m.). The success of the methods used in producing directed radio beams makes it seem unlikely that the deviation can in any case be very great. It is probably very nearly correct to assume, therefore, that when the line *AB* is directed toward the station both receiving stations are in the line of propagation of the waves, while if *AB* is perpendicular to this direction the two stations lie in the same wave front.

Since it is a fundamental characteristic of a wave front that the phase is the same at all points, we should expect, if *A* and *B* are in the line at right angles to the station direction, and therefore presumably in the wave front, that the oscilloscope figure would indicate a constant phase relation between the two signals. In other words the figure, after being adjusted to a straight line as indicated above, should remain a straight line. During the day time, with the loops set to receive the ground wave (i.e. with the plane of each loop directed toward the sending station) the oscilloscope figure usually did remain unchanged for long periods. It was not unusual, however, for the line to tilt back and forth through a small angle. And even in the day time the line sometimes opened up to form a narrow ellipse. At night the changes in the figure were usually more marked and more erratic.

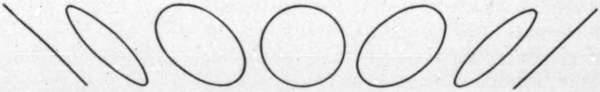
It was noticed that when the figure indicated equality of phase and amplitude at the two loops the sky wave was either absent or small; and that the oscilloscope figure showed large deviations from a straight line only when the sky wave was of considerable intensity and subject to large fluctuations. Since a loop set to receive the ground wave will respond also to the vertically polarized component of the sky wave, it seems not unlikely that if it were possible to eliminate the disturbing effect of the sky wave the ground wave alone would be found to behave in a perfectly normal manner.

If the loops were set to receive the sky wave, observations during the day were difficult because of the small intensity of the signals. At night the signals were of ample intensity but the figure rarely showed any close approach to constancy either in shape or tilt. On frequent occasions the changes in the figure were so great and so rapid that it was difficult to tell from its appearance in which quadrant the original straight line adjustment had been made. In numerous instances the figure has been observed as a horizontal or vertical line—in other words the signal was zero at one loop at the same time that it was of normal strength at the other—only one third of a wave-length distant! On these same occasions phase differences as great as 180 degrees were frequently observed. It is clear that the propagation of the sky wave is so profoundly affected by the atmospheric conditions that it cannot ordinarily be regarded as a single wave train.

Observations made with the two loops in the line of propagation lead to similar conclusions. In these observations also the loops were at a distance apart ranging from $\lambda/4$ to λ—usually about $\lambda/3$.

During the day time, and at night also in case the sky wave was of small intensity, the behavior of the oscilloscope figure, when the loops were in the line of propagation and set to receive the ground wave, was such as would be expected. If adjusted to a straight line at 45 degrees the figure remained unchanged for long periods. If the adjustment was made when the loops were close together and one loop, carried by a little cart, was then moved to a greater distance from the station, the figure went through the sequence shown in Fig. 11. To complete this sequence for station WGY (λ = 380 m.) required that the distance between the loops had been increased to 175 m. instead of to 190 m., as would be expected. The conditions did not remain steady long enough for this observation to be repeated. It seems to us probable that the difference is due to the fact that the conditions were not in reality entirely steady. Whenever the sky wave was of considerable intensity the behavior of the figure became erratic. There was every indication, however, that the disturbance was due to the vertically polarized component of the sky wave and that the behavior of the ground wave alone was still normal.

FIG. 11.

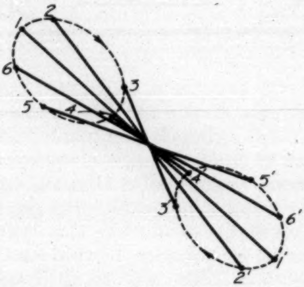


With the two loops at a distance of from λ/4 to λ apart in the line of propagation, and set to receive the sky wave, there was always evidence of changes in the relative amplitude and phase at the two loops, and the changes in the oscilloscope figure were often so rapid and erratic as to be quite beyond description. On one occasion only were the conditions steady enough to make it seem worth while to observe the change in phase resulting from a change in the distance between the loops. It was found that the phase difference increased less rapidly with the distance of separation than when the loops were set to receive the ground wave. This is to be expected in the case of waves approaching the earth obliquely. But the conditions were not sufficiently steady to permit a reliable estimate of the angle of incidence.

While in the case of most of our observations of differential fading the behavior of the oscilloscope figure was too erratic to permit of interpretation, there were occasional instances of systematic and relatively simple behavior. For example, the form of differential fading that is illustrated in Fig. 12 was frequently observed when the two loops were in the wave front and set for the ground wave. The figure remained a straight line, except occasionally when the amplitude was very small; but this line changed continuously in length and inclination, and in such a way as to make the upper end of the line move along a rather narrow ellipse, occupying successively the positions 1, 2, 3. . . . At the same time the lower end of the line moved around the symmetrically placed lower ellipse 1′, 2′, 3′. . . .

On one occasion (July 23, 1930, Station WHAM) when this type of fading continued from 7:41 P.M. to 7:47 P.M. a complete cycle at first occupied about 4 sec.; then the movement became slower and the amplitude changes less marked. In the course of six minutes there were several reversals in the direction of the motion, i.e. from the sequence 1, 2 . . . 6, to the sequence 6, 5 . . . 1 and back again.

FIG. 12.

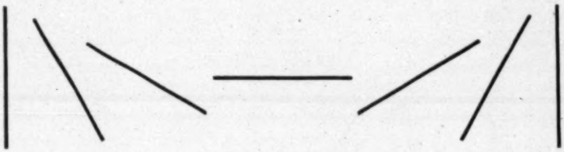


At 7:47 both loops were turned into the plane of the wave front so as to respond to the sky wave only. The figure was then found to rotate continuously, at the same time changing back and forth from a line to an ellipse and expanding and contracting, and with occasional reversals of the direction of rotation. A little later the figure shifted back and forth

through the same sequence as that shown in Fig. 11. This indicates a progressive change in the phase difference between the waves at the two loops, with no change in amplitude. In other words, if the electric vector at loop A is $E_0 \cos 2\pi nt$ that at loop B is $E \cos (2\pi nt + \varphi)$ where $\varphi = at$. In other words if the frequency of one wave is n that of the other is $n + a/2\pi$. The sequence of Fig. 11 thus indicates a sort of Doppler effect, which may be explained on the assumption that the two waves reach the receiving loops by different routes and that the effective lengths of the two paths are changing at different rates.

On another occasion (Aug. 28, 1930, 5:45 P.M., Station WGY) with the loops 200 yds. apart in the line of propagation and both set to receive the sky wave, the figure was observed to be a straight line of nearly constant length which rotated continuously (see Fig. 13) making one complete turn in about 15 sec. In this case if the electric vector at one loop is $A \cos pt$ that at the other loop must be $B \cos pt$ with $A = E_0 \cos nt$ and $B = E_0 \sin nt$. $n = 2\pi/15$ sec.$^{-1}$ This might be interpreted as indicating that the waves received at the two loops were out of phase by 90 degrees and each subject to a slow modulation.

Fig. 13.



A more plausible explanation of this particular case, and one which seems to be applicable to all of our observations, is, however, the following:

Unless the atmospheric conditions are remarkably steady we must expect considerable variations in the ionic density from point to point and from instant to instant. Probably the ionic density is never a function of the height alone, but depends also upon the time and the position in the horizontal plane. The condition is one of turbulence, quite incapable of description in analytical form. It seems not unreasonable to draw an analogy between the electrical conditions in the upper atmosphere and the conditions in irregularly heated air. The radio waves will be influenced by these conditions in much the same way that light is influenced in passing over a hot stove. In the case of radio waves we have as an additional complication the effects of magnetic double refraction and rotation. The result will be that where we might expect a simple wave front we find instead a complicated interference pattern, which, if the conditions are steady, will consist of a fixed system of interference bands. It is quite possible that there should be additive interference at one receiver and destructive interference at the other. If we could investigate the conditions at different points in the interference pattern of a Michelson interferometer, or at the focus of a lens, we should not expect to find constancy of either amplitude or phase. If an interferometer were mounted above a hot stove the bands would probably disappear. At each instant there would still be a definite interference pattern—although probably a very complicated one. But the pattern would change so rapidly that in the case of light it could not be observed. In the case of radio waves everything is on a larger scale—even time—so that it is possible to follow the changes from instant to instant.

According to this view the sequences which show evidence of system are to be found at times when the electrical conditions of the atmosphere are changing in a relatively simple way and, for the time being, at a fairly constant rate. The sequence of Fig. 13, for example, may be accounted for if the interference pattern consists of a series of equally spaced bands which are slowly drifting in the direction of propagation, so that a band which was at first at loop *A* will be a little later at loop *B*. Such an effect might be produced by a drifting cloud of ionized air, or by a steady increase or decrease in the ionization of the stationary atmosphere. The phenomena exhibited in Fig. 13 form perhaps a rough electrical analogue of the shadow bands seen at the time of an eclipse. It seems probable that a good deal can be learned by a study of differential fading at near-by points. Simultaneous observations, for example, with stations in the line of propagation and in the wave front might enable us to determine the speed and direction in which the assumed ionized clouds are moving.

Observations of differential fading with short waves have been begun, but the amount of data accumulated is not yet sufficient to draw general conclusions. The absence of a ground wave in the case of short waves in some respects simplifies the interpretation of the observations. But on the other hand the experimental difficulties are greater. Observations with both short waves and waves in the broadcast range will be continued.

X-RAYS IN THE STEEL INDUSTRY.[1]

BY

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The present century now only thirty years old has witnessed phenomenal changes in science. The Electron Theory, The Quantum Hypothesis, The Bohr-Sommerfield atom, modern developments of radiation theory have revolutionized our concepts of the physical universe. The rapidity with which our scientific knowledge is growing is marvelous. Great advances have taken place in industry during this period. Buildings, bridges, machines, plants, have undergone tremendous changes. The tendency is toward greater size, greater complexity, greater power, greater output. The rapidity with which these developments have occurred is astounding. The vastness of the changes, the greatness of their effects on economic life can hardly be visualized. Fifteen years ago 300 lbs. per square inch was considered high steam pressure. To-day 1,200 lbs. per square inch is common and 3,000 lb. pressures are being tried experimentally. Fifteen years ago 50,000 K.W. generators were dreamed of, but none had been made. To-day there is in construction a turbine that will take steam at 1,200 lbs. per sq. in. pressure, and discharge it into a vacuum and that will drive a generator delivering over 200,000 K.W. The magnitude of this industrial machine may be realized better when we consider that the average load in Metropolitan Boston to-day is around 200,000 K.W. This single machine will be capable of delivering electrical energy to supply the needs of a city of 1,000,000 people. Illustrations of colossal growth could be selected from railway transportation, from chemical manufacturing, from oil refining, from most of the fields of economic production.

It is not accidental that parallel development occurs in science and industry. The two fields of human endeavor are related. Each responds to stimulus from the other. Probably neither could develop without the other. Should progress in the development of the machines of industry cease, stagnation in the progress of scientific thought would follow, and the evolution of civilization would be interrupted. It is vital to society that this progress should not stop and that industry continue its upward and onward trend.

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[1] Released for Publication by Authority of the Ordnance Dept., U. S. Army.

The engineer is the builder for industry. From a sure base of definite achievement he erects in his mental vision greater edifices, more powerful mechanisms. The concrete realization of his dreams requires building materials, the more important of which are metals, chiefly steels. For these he turns to the metallurgist. Consequent to the rapid development of modern industry there has been a rapid development of alloy steels. There has been, and is, an ever increasing demand for steels of higher and higher physical properties, and for steels of specialized properties. This demand has been met by studies to improve processes and to improve chemical compositions. Much of the investigative work has been purely empirical, partly because of the fact that in spite of the millions of tons of steel in daily use, its fundamental nature has not been well understood. In fact certain microstructures in steel are to-day matters of controversial argument. This lack of understanding of the nature of steel is no reflection on the metallurgists who have devoted years to its study, it has been due for the most part to the absence of adequate methods of study. Steel is very heterogeneous and highly complex. It owes its intrinsic physical properties to microscopic and to submicroscopic structures within the metal.

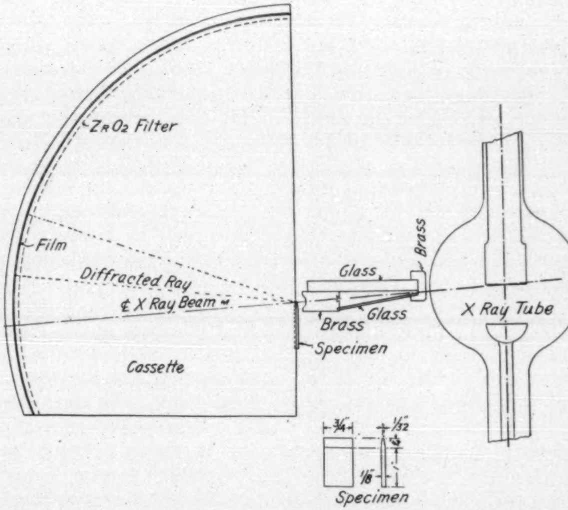
There has been developed in recent years new tools of research that will permit the study of these fine structures. The microscope in the hands of F. F. Lucas has reached hitherto unexplored regions in the field of microscopy. Even his remarkable achievements, however, have failed to picture the finest structures in steel. There has been needed an instrument of even greater resolving power. Such an instrument is found in the X-ray diffraction apparatus. This tool makes possible the charting of relative spatial arrangements among atoms themselves. X-ray studies of metals have been so fruitful during the past few years that the American Society for Testing Materials has recognized a new branch of metallurgical science, X-ray Metallography.

The empirical results of applied research are reflected quickly in modified manufacturing technique, and in the development of new products. Such research serves industry directly. The more fundamental theoretical research may not serve directly in the solution of production problems. It aims to increase the sum of knowledge regarding basic facts and principles. It serves pure science. The X-ray diffraction method for the study of metals comes within the category of fundamental rather than applied research. Its more important achievements do not affect production directly, but do supply much needed basic information that will guide and direct applied research. To illustrate, X-ray diffraction studies do not give specific information about the hardening of copper, but they do give very definite information as to the nature of hardness. With this information, it is possible and probable that copper will be hardened as effectively as aluminum has been. The actual hardening of copper will be a problem in applied research.

In the hands of such men as Hull, Davey, Bain, Jefferies, Archer, McKeehan, Clark, and a host of others in this country and abroad, this new tool has already produced remarkable results in clearing up the mystery that has enshrouded steel. We know now the nature of hardness. We know why steels are strong or weak. We have a clear picture of a solid solution. We know how metals flow in plastic deformation. A few tenths of one per cent. of carbon added to iron changes iron, a relatively weak substance to steel, our strongest building material. This fact has been known for centuries. We can state pretty definitely now the exact rôle of the carbon atom in this transformation, and the part it plays in the quenching and tempering of steel.

Such a powerful tool deserves description. Figure 1 shows one arrangement of X-ray tube, metal specimen, and photographic film. The X-ray beam is passed, in this case, across the surface of the metal. The rays are diffracted from many small crystals in the metal and diffracted beams from different atomic planes in the crystals are registered at different points on the photographic film. Figure 2 is a drawing taken directly from developed photographic films, showing typical X-ray crystal spectra from two steels. The diagram shows the arrangement of the iron atoms in the crystal patterns. This pattern was worked out from measurements of the relative positions of the spectrum line. The P_2J was found to have a slightly enlarged crystal unit, due presumably to the presence of an alloying constituent that was held in solid solution in the iron. Figure 3 illustrates spectra from three different steels. One is a plain carbon steel, one a 13 per cent. manganese steel, and the other a tool steel. The first line marks the position of the undeviated beam, the other lines represent images from the rays diffracted by differ-

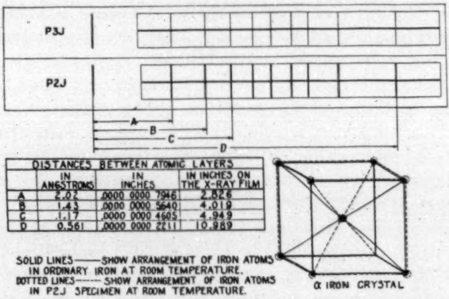
FIG. 1.



Hull diffraction tube slit system, and diffraction camera. X-rays proceed from the anode of the X-ray tube. After passing through a slit system that constructs the rays into a flat beam, they pass over the surface of a metal specimen. The minute crystals in the surface reflect the part of the beam at different angles according to the different plane systems in the crystal. The reflected rays pass through a filter and are caught on a photographic film that is bent around a cylindrical surface that has the surface of the metal specimen in its axis.

ent atomic planes. Measurements of these positions relative to the undeviated line give data that permit the construction of the crystal patterns. Some idea of the magnitudes in-

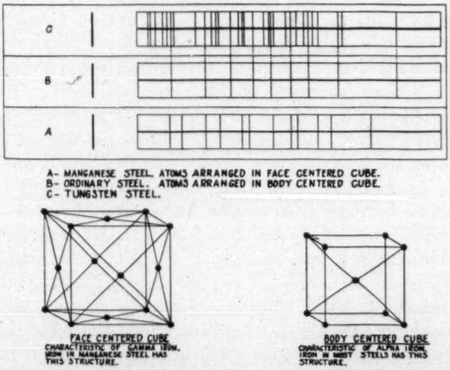
FIG. 2.


P3J
P2J
A
B
C
D
SOLID LINES——SHOW ARRANGEMENT OF IRON ATOMS IN ORDINARY IRON AT ROOM TEMPERATURE.
DOTTED LINES----- SHOW ARRANGEMENT OF IRON ATOMS IN P2J SPECIMEN AT ROOM TEMPERATURE.
α IRON CRYSTAL

DISTANCES BETWEEN ATOMIC LAYERS			
	IN ANGSTROMS	IN INCHES	IN INCHES ON THE X-RAY FILM
A	2.02	.0000 0000 7946	2.826
B	1.43	.0000 0000 5640	4.019
C	1.17	.0000 0000 4605	4.949
D	0.561	.0000 0000 2211	10.989

Drawings made from two films showing typical X-ray crystal spectra. The heavy line to the left is the image of the undeviated beam. The lighter lines represent reflections from different atomic plane systems in the crystal. The table of measurements for four of the lines gives the relationship between the film magnitudes and the corresponding magnitudes within the metallic crystals. This figure illustrates also a practical application of diffraction data. The P$_3$J specimen showed peculiar physical properties. This was traced to a different sized crystal pattern presumably due to the fact that the P$_3$J specimen contained an impurity that was in solution in the iron.

FIG. 3.


C
B
A
A- MANGANESE STEEL. ATOMS ARRANGED IN FACE CENTERED CUBE.
B- ORDINARY STEEL. ATOMS ARRANGED IN BODY CENTERED CUBE.
C- TUNGSTEN STEEL.
FACE CENTERED CUBE
CHARACTERISTIC OF GAMMA IRON. IRON IN MANGANESE STEEL HAS THIS STRUCTURE.
BODY CENTERED CUBE
CHARACTERISTIC OF ALPHA IRON. IRON IN MOST STEELS HAS THIS STRUCTURE.

Drawings from diffraction films. Crystal spectra from three different steels. The atomic arrangements in the crystal patterns worked out from films *A* and *B* are shown drawn to scale.

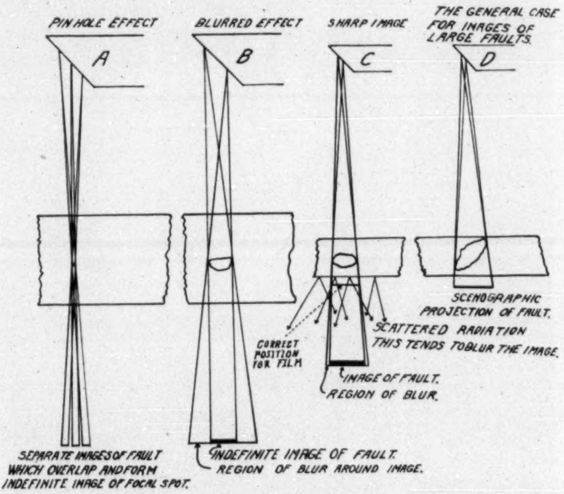
volved may be gained from a study of the table in the figure. With a good film different readings of the line positions can be duplicated within about ± .010 inch, for a line corresponding to that marked *D* in the figure. The error in measurement is approximately ± .1 per cent. The 10.989 inches on the film converted to interatomic spacings within the crystal means .0000000221 inch or .561 angstrom unit (1 angstrom = 1×10^{-8} cm.). This measurement is certain to ± .1 per cent. that is to ± .005 angstrom unit. An iron atom has a diameter of 2.48 Å. This particular measurement is in error by approximately ± 1/500 of an atom diameter. It is interesting to compare these magnitudes with those obtained by microscopic examination. F. F. Lucas obtains useful magnifications up to 4,500 diameters. That is, he can distinguish distances on the ground glass of the camera corresponding approximately to 1×10^{-5} inches in the metal itself. Under favorable conditions, as illustrated in Fig. 3, the X-ray diffraction method will distinguish distances as small as 5×10^{-11} inches. That is, the X-ray in this sense is 100,000 times more sensitive than the microscope. Such a tool is worthy of respect.

In the above it has been attempted merely to illustrate some of the possibilities of this method of study. A complete discussion could not be attempted within the limits of this paper. The Hull-Debeye type of diffraction spectra shown in Figs. 2 and 3 is only one of the kinds of diffraction patterns that is obtainable. Another that promises to be equally important is the Laue pinhole pattern developed in this country largely by Prof. G. L. Clark of the University of Illinois.

Steels of greater and greater intrinsic merits are required to carry the modern loads. X-ray studies, such as we have described, permit understanding and eventual control of the fundamental factors that make steel strong and durable. But when the new steels are developed their superior properties can be realized in practice, only if the structural units made from them are sound. No forging or casting is stronger than its weakest vital section. Cracks, nonmetallic inclusions, blow holes, etc., must be eliminated in order that those units may render in actual use the service of which the metal is inherently capable.

There is an additional reason for the elimination of gross defects in steel building units. The urge that drives industry toward ever greater manufacturing units is economic. It must have not only greater output, but also greater efficiency of output. The colossal machines of to-day demand not only good steel, but complete assurance of dependable service in the units made from that steel. A 200,000 K.W. generator cannot be shut down to replace a broken casting or other structural unit. It costs too much to interrupt that amount of production even for a moment. The casting or forging *Must Not Fail.*

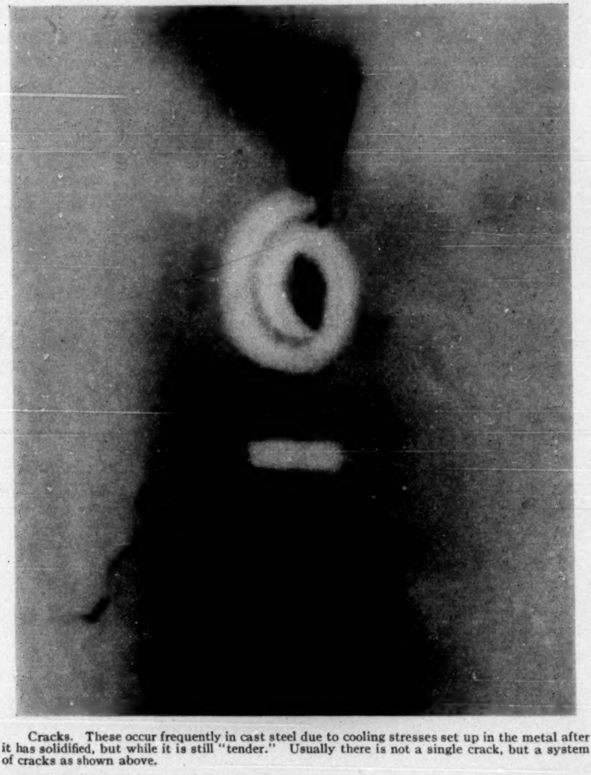
FIG. 4.


PIN HOLE EFFECT
BLURRED EFFECT
SHARP IMAGE
THE GENERAL CASE FOR IMAGES OF LARGE FAULTS.
A
B
C
D
SCENOGRAPHIC PROJECTION OF FAULT.
SCATTERED RADIATION THIS TENDS TO BLUR THE IMAGE.
CORRECT POSITION FOR FILM
IMAGE OF FAULT.
REGION OF BLUR.
SEPARATE IMAGES OF FAULT WHICH OVERLAP AND FORM INDEFINITE IMAGE OF FOCAL SPOT.
INDEFINITE IMAGE OF FAULT.
REGION OF BLUR AROUND IMAGE.

X-ray photography. Schematic diagrams showing how defects in steel are photographed by X-rays. *A*, *B*, *C*, and *D* represent the anode of an X-ray tube. The cavity acts really as the aperture of a pin hole camera. When it is very small the image formed is really that of the active area of the anode surface. When it is larger an image of the cavity results.

Here again the X-ray is of service. Castings and welded structures have in the past been peculiarly subject to gross defects. These defects hidden within the metal may be photographed by means of X-rays, in much the same way that the physician photographs a broken bone, or the dentist X-rays defective teeth. Figure 4 illustrates how such pictures may be obtained. Figures 5, 6, 7 illustrate typical X-ray negatives of casting defects. At present commercially available apparatus can be used for practicable application

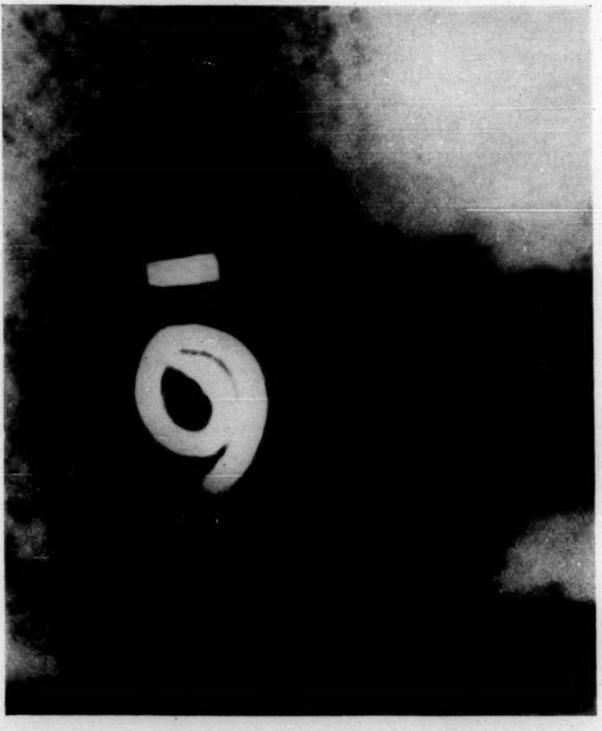
FIG. 5.



Cracks. These occur frequently in cast steel due to cooling stresses set up in the metal after it has solidified, but while it is still "tender." Usually there is not a single crack, but a system of cracks as shown above.

of the method to steel sections up to 3 or 3½ inches in thickness. To get pictures through this thickness requires potentials on the X-ray tube of around 250,000 volts. Dr. Coolidge is developing a new tube that can be operated at potentials of 900,000 volts. This tube will increase greatly the thick-

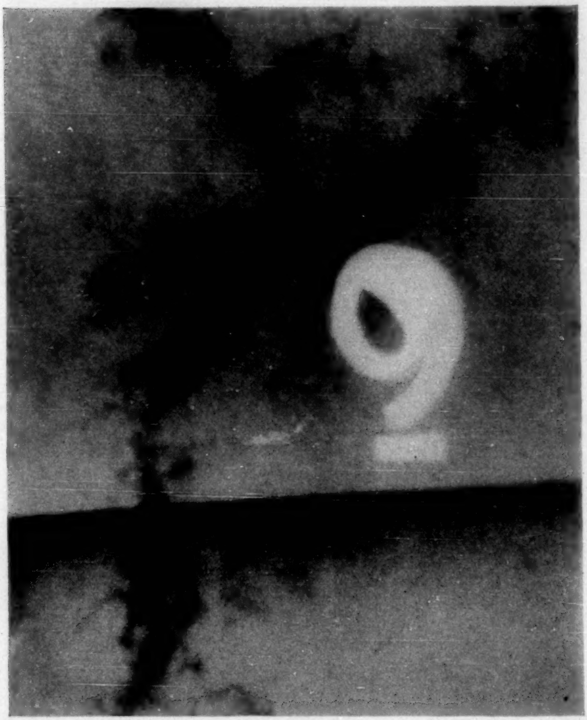
FIG. 6.



Spongy metal. Metal occupies more volume in the liquid state than in the solid. When it solidifies a definite cavity may be formed (see Fig. 7), or there may be a region that contains many minute cavities as in the case illustrated here.

ness of metal that can be radiographed. At the Naval research laboratory Drs. Mehl, Dean, and Barrett have been

FIG. 7.



Pipe. Molten metal is fed into the casting while it solidifies. Sometimes feeding is imperfect and pipes occur, either as definite cavities as shown above or as spongy regions illustrated in Fig. 6.

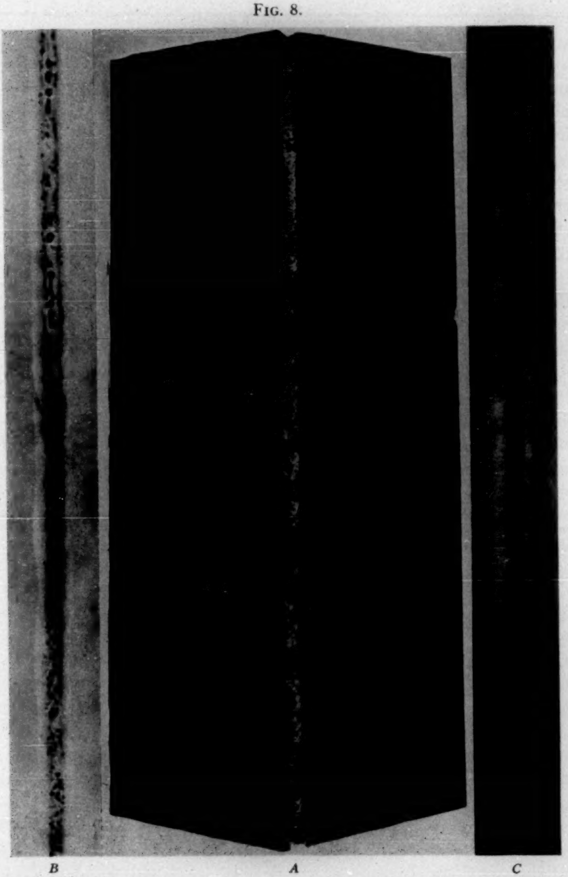
successful in using gamma rays from radium in photographing gross defects in ten inches of steel. This development is a very important one and may supplant X-ray methods for examining heavy sections, though probably it will not do so for light ones.

The radiographing of gross defects in castings has been used extensively in recent years as an inspection method for eliminating unsound units. It has been used more economically to develop manufacturing technique that will produce sound castings. In the past few years the development of the art of welding, particularly electric welding, has given the engineer a new form of construction. In many fields castings have been supplanted by formed and welded steel plates. The time is approaching when the riveting of steel buildings with its accompanying noise will be discontinued in favor of welding. But the welds will have to be completely dependable.

Radiographic tests are now being used for searching out defects in welds as they are for the examination of castings. Figure 8*a* shows a photograph of a weld that from visual inspection seemed to be a remarkably good one. Figure 8*b* shows the X-ray picture of this weld. It was a remarkably poor one. Figure 8*c* shows the conditions after developing the welding technique under X-ray control. Incidentally it may be remarked that the weld from which 8*c* was taken resembled the first one so closely that it would have been difficult to distinguish them by visual examination.

This application also is used most economically to develop technique. It is better to make good welds than to reject bad ones. The X-ray control of welding practice, however, differs somewhat from the similar control of casting practice. In the latter case, such factors as proper size and location of gates and sinkheads, proper core strength, proper venting of sand molds, etc., are sought. These are fixed conditions, and once determined are not subject to variation. In the case of electric arc welding, the factors sought are proper size and type of welding rod, proper angle of holding the electrode, proper current and voltage in the arc, etc. Most of the factors are not fixed conditions, but are subject to variation by the operator during the operation. For this reason X-ray control of welding technique is of more personal application. It forces the operator to be acutely conscious of his work at all times, and to give it the closest attention. It develops

FIG. 8.



(*Army Ordnance Vol. XI, No. 62, p. 124.*)

Development of welding technique at Watertown Arsenal. *A.* Photograph of a weld that judged by visual inspection was particularly good. *B.* X-ray picture through this weld. The black spots represent gas cavities. Judged by X-ray inspection the weld was particularly poor. *C.* X-ray picture taken after development of proper technique. This weld resembled the first one so closely that it would have been difficult to distinguish them by a visual examination. The metal of the weld was found to be sound. The weld was nearly perfect.

the welder. This is of great importance. It is not practicable, at present at least, to test the welds of a steel building other than by visual inspection. This inspection, however, is sufficient if it is backed up by a knowledge of the welding rod, the current and potential used, and a knowledge of the ability, and the character of the welder.

In the foregoing, the writer has attempted to point out in a general way that X-rays have become exceedingly important to the steel maker and user. Industry, which is vital to our civilization, depends on steel. It has been erected on a foundation in which steel is of dominant importance. The skeleton of its super structure is steel. Its future development depends on steel or a hypothetical steel substitute. Steel must be improved to meet the growing demands of a new age. This improvement, as far as it is possible to predict the future, will result largely from fundamental information that has been or will be obtained by means of X-ray diffraction studies. It is one thing to produce good metal, it is another to use it most efficiently. The efficient use of steel is as important as the making of it. Here too the X-rays function. By means of radiographic tests the engineer and the ultimate user may be assured that the intrinsic merits of the metal will be realized in actual performance.

A Determination of e/m for an Electron by Direct Measurement of the Velocity of Cathode Rays. CHARLOTTE T. PERRY AND E. L. CHAFFEE. (*Phys. Rev.*, Vol. 36, No. 5.) Though many methods of measuring the ratio of the electrical charge of an electron to its mass when at rest have been used they fall into two classes, i.e. (*a*) those made with electrons free from atoms and molecules such as cathode rays, photo-electrons and β-particles, and (*b*) those in which the electrons were within atoms, such as used the Zeeman effect and spectroscopic data. The two methods do not give identical values for the ratio which has become one of the most important of physical quantities. The first gives 1.7689×10^7 abs. em. units and the second 1.7608×10^7 units.

In the apparatus employed by the authors cathode rays are projected by the voltage of batteries along the axis of a vacuum tube placed parallel to the earth's magnetic field. After going through a small hole in a diaphragm they pass between two plates parallel to the axis of the tube. These plates are joined to a high frequency oscillator. Those rays which traverse the alternating electrostatic field between the plates when its value is zero are not deflected but continue along the tube with no change of direction. Rays arriving at other times are deflected and eliminated by screens. Seventy-five cm. farther along in the tube is a second pair of plates joined likewise to the oscillator but the two electric fields have a phase difference of 180°. Only the undeflected rays traverse the second field. If they reach it when its value is zero they again pass without deflection and make a spot on a fluorescent screen. The condition for absence of deflection as shown by a centrally located spot is that the electrons travel the distance between the two fields in half a cycle of the oscillator, or in any multiple thereof. The oscillator was kept at constant frequency by comparison with an oscillating crystal of frequency 1,680,890 cycles per sec. The location of the spot was adjusted by varying the voltage applied to the electrons before reaching the first field. The velocity of the electrons ranged from $.6 \times 10^{10}$ cm/sec to $.8 \times 10^{10}$ cm/sec.

The computed average value of the ratio is $e/m = 1.761 \times 10^7$ abs. em units. This agrees very closely with the results previously obtained by using electrons inside of atoms, and not as might have been expected with the result coming from such free electrons as were used in the arrangement from which the result was obtained.

G. F. S.

RECENT DIELECTRIC CONSTANT THEORY AND ITS RELATION TO PROBLEMS OF ELECTRICAL INSULATION.*

BY

JOHN WARREN WILLIAMS, Ph.D.

Modern physics teaches that electricity is a fundamental constituent, if not the whole content, of all matter. The negative electricity consists of electrons or individual atoms of electricity, which always are of uniform mass (9.04×10^{-28} gr.) and of like charge. They are assumed to be the carriers of electricity. Positive electricity, on the other hand, differs in that there is no evidence for the existence of carriers of less than atomic mass. These positive charges are now assumed to be multiples of a unit, the hydrogen atom, which carries a charge equivalent to that of one electron. In other words, electricity and matter consist of protons and electrons, the former being charges of positive electricity and the latter of negative electricity. However, while electrons which have been dislodged from atoms may pass readily from one body to another, it is found that the protons are associated with atoms and have a mobility of an altogether different order of magnitude.

Molecules are composed of these positively and negatively charged particles in such numbers that they are ordinarily neutral as a whole. Such a body may become charged in many ways, and is defined simply as one in which there is an excess (or deficit) of negative electrons. The distinction between one type of insulator and conductor as regards the phenomena of induction and conduction depends essentially upon the fact that in the conductors there is a large number of the free dissociated electrons which can be pulled about from one part of the medium to another under the action of forces exerted from other electrified bodies, while in insulators the number of these free electrons is comparatively small. Any process which will reduce the number of free electrons will reduce the conductance of a material, thus certain forms of sulphur and selenium are poorer conductors in the dark than they are in the light because electrons are freed by the latter. It appears, therefore, that in order for a substance of this kind to be a good insulating material it is necessary to keep the number of free electrons as low as possible.

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* Presented at the Third Annual Meeting of the Committee on Electrical Insulation of the National Research Council at Washington, D. C., November 7, 1930.

There are many insulating materials in which dielectric absorption and dielectric loss are believed to depend upon the mobility and adsorbability of ions which happen to be present. Also, it has been suggested in a number of places that absorption and loss in other dielectrics are much more complex than such mechanisms would lead one to believe, and that even if there were no free electrons or ions present a loss could be due to the dielectric polarization and orientation of the neutral molecules themselves. Joffé[1] and his students insist they have found that if the conductance is properly measured it explains all the dielectric loss, except that obviously due to inhomogeneity of the dielectric in the Maxwell-Wagner sense, but this point of view has been adversely criticized by Smekal[2] and many others.

Different investigators seem agreed that no substance is a perfect insulator. The best insulators are amber, transformer oil, glass, quartz, air, silk, certain of the synthetic resins, and rubber. Dry wood, paper, and cotton are less efficient. If one stops to think how much actually is known concerning the structure of these materials it is not so strange after all that various investigators are not agreed as to the mechanism of dielectric loss. If there are different mechanisms by which such a loss may result one may operate in a crystalline dielectric at a high frequency and low temperature and in the absence of moisture while another may contribute practically the entire loss in an amorphous material at a low frequency and in the presence of considerable moisture. Of the dielectrics mentioned some are definitely crystalline in nature, but it seems as if the majority would have to be considered as amorphous and moisture absorbing materials. I would like to emphasize, therefore, the importance of the study of dielectrics from the standpoint of their actual structures. We must learn about the sizes and shapes of the individual molecules, we must learn about the manner in which these molecules are aggregated to form particles, and we must discover the manner of arrangement of these aggregates. The first is a problem of molecular physics while the others are problems of colloid chemistry. Definite progress is now being made in these directions, indeed so much is being done that I could hardly hope to outline it completely in the space at my disposal. Its importance does seem to justify the detailed consideration of certain of the methods in this paper.

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[1] Joffé, *Z. Physik*, **62**, 730 (1930); "The Physics of Crystals," McGraw Hill Book Co., New York, 1928.

[2] Smekal, *Z. techn. Physik*, **8**, 561 (1927); *Z. Physik*, **45**, 869 (1927).

THE STRUCTURE OF MOLECULES.

There have been made available a number of methods by which the sizes and shapes of molecules may be studied. Among them may be mentioned:

1. The relationship between surface tension and temperature.
2. Studies of crystals by means of X-Rays.
3. Electron and X-Ray interference measurements with single molecules and with liquids.
4. Analysis of infra red and Raman spectra.
5. Variation of dielectric constant with temperature and frequency.

In view of the fact that the bulletins which have been sent to the various members of this committee have contained numerous references to the dipole theory of Debye a discussion of some of its cardinal points seems pertinent. No attempt will be made to cover the already vast literature of this subject, rather the purpose will be to indicate the type of information which it may make available by a careful consideration of a few specific cases.

The characteristic feature of a polar molecule is a distance of separation between the centers of gravity of the two equal and opposite electrical charges. The polarity of a molecule becomes less as these centers of gravity in a molecule approach each other. A molecule in which there is a zero distance between the centers of gravity is non-polar. The electric moment of a molecule, defined as the product of the charge by the distance of separation between the centers of positive and negative electricity, is a quantitative measure of the polarity of a molecule. When a molecule possessing an electrical dipole is placed in an electrostatic field it will be affected in two ways by it. There will be a polarization by orientation, and there will be a polarization by deformation. Both polarizations will contribute to the dielectric constant. They are defined in the following way: All the molecules will *tend* to set themselves in a direction in opposition to the field, just as the elementary magnets do in a magnetic field according to the so-called molecular theory of magnetism, except in the latter case it is not assumed in the theory that the elementary magnets are molecules in the chemical sense. In the electrical case we cannot expect to get complete orientation for several reasons—the tendency to orient will be opposed by thermal agitation and by the influence of the field of adjacent dipoles, and the field intensity necessary to produce such a saturation will ordinarily be considerably higher than the dielectric will stand. In addition to this orientation effect the molecules will be distorted because of the effect of the field in displacing its atoms and electrons.

It is found that the polarizability, α, the quantity which relates the average moment of one molecule, m, with the actual electrical intensity acting upon it, F, according to the equation

$$m = \alpha F \tag{1}$$

may be represented by a formula of the type

$$\alpha = \alpha_0 + \frac{\mu^2}{3kT}, \tag{2}$$

where α_0 is a constant factor which takes care of the distortion effect

and $\frac{\mu^2}{3kT}$ is a factor, dependent upon the magnitude of the electric moment, μ, the absolute temperature, T, and a numerical quantity. This factor takes care of the orientation effect.

For practical purposes it is convenient to express this equation in terms of the molar polarization, P, by multiplying by the universal factor $\frac{4\pi}{3}\cdot N$. Thus,

$$P = \frac{4\pi}{3} N\left(\alpha_0 + \frac{\mu^2}{3kT}\right). \tag{3}$$

The molar polarization can be determined quite readily, if one is willing to assume the validity of certain assumptions concerning the internal field made by Clausius and Mosotti, by measuring the dielectric constant, ϵ, and density, d, for suitable systems. The Clausius-Mosotti equation is as follows:

$$P = \frac{\epsilon - 1}{\epsilon + 2} \cdot \frac{M}{d}. \tag{4}$$

Measurements of the dielectric constant of gases and vapors are important because in this case there is no doubt that the molar polarization can be calculated by the formula

$$P = \frac{\epsilon - 1}{3} \cdot \frac{M}{d}. \tag{5}$$

To determine whether or not a molecule is polar it is simply necessary to measure the dielectric constant and density at different temperatures. If P, calculated from Equation 5, is independent of temperature the molecule is non-polar, that is, its electric moment is zero. On the other hand, if P depends upon temperature the molecule is polar. Equation 3 indicates that if P is plotted as a function of $1/T$ a straight line should result. Expressing the experimental results by means of the formula

$$P = a + \frac{b}{T}.$$

the electric moment of a molecule can be calculated because it can be shown that

$$b = \frac{4\pi}{3} N \frac{\mu^2}{3k}.$$

Such measurements have now been made by a number of investigators.

Unfortunately the electric moment of every molecule cannot be determined in this way because many substances cannot readily be obtained in the vapor state. It is possible, however, to study the dielectric constant and density of dilute solutions of polar molecules dissolved in a solvent which is known to be non-polar and feel reasonably sure that the Clausius-Mosotti equation will give the total polarization, since the only interaction of importance will be between the polar molecules and the surrounding non-polar molecules of the solvent. In this way measurements of the polarity of a molecule become actually very simple. In the case of such a solution Formula 4 assumes the form,

$$P_{1,2} = \frac{\epsilon - 1}{\epsilon + 2} \cdot \frac{f_1 M_1 + f_2 M_2}{d} = f_1 P_1 + f_2 P_2, \tag{6}$$

where $P_{1,2}$ = molar polarization of the solution,
f_1, f_2 = mole fraction of solvent and solute, respectively,
M_1, M_2 = molecular weight of solvent and solute, respectively,
and P_1, P_2 = molar polarization of solvent and solute, respectively.

Since the molar polarization of the solvent may be assumed to be always proportional to its mole fraction in solution the molar polarization of the solute, or molecule whose electric moment is sought, may be determined as a function of its concentration. We are directly concerned in the present discussion only with the limiting value of P_2 which is reached in infinitely dilute solution, because this represents the molar polarization of a free polar molecule not in interaction with other polar molecules of any kind whatsoever. It is just this extrapolation which makes permissible the use of the Clausius-Mosotti law in these particular solutions.

As a matter of fact it is possible to obtain a quite exact result for the electric moment of a molecule by making dielectric constant and density measurements for these dilute solutions at a single temperature because the polarization due to deformation of the polar molecule may be separated from the total polarization by means of the Lorentz-Lorenz formula. This separation involves the use of an alternating field of sufficiently high frequency. Under such conditions, the molecules will not have time to set themselves parallel to the field, and a smaller value will be found for the dielectric constant, in which only the distortion effect comes into play. Light waves are ordinarily used as the alternating field, so that the quantity actually measured is the refractive index. It is necessary to use the refractive index for infrared radiation, because if visible light is employed the vibrations are so rapid that only the electrons can be distorted and the contribution of the atoms will form a part (ordinarily a relatively small one in the case of a polar molecule) of the remainder, that is of the apparent polarization due to orientation.

One is not surprised at the intimate connection between electric moment data and the structure of molecules because after all the electric moment measures the symmetry, or lack of symmetry, of that fundamental constituent of matter, electricity. Thus, if we start with the ordinary picture of the carbon atom with its four valences directed toward the corners of a regular tetrahedron we expect that methane and carbon tetrachloride will be non-polar, not because of the absence of polar linkages or polar atoms but because of the resultant electrical symmetry. The validity of the assumption concerning the direction of the four valences is indicated by the fact that these molecules are found experimentally to be non-polar. One can likewise draw important conclusions concerning the structure of benzene from a study of the electric moment of this molecule and some of its simple derivatives. The fact that certain para-disubstituted benzenes and certain symmetrically tri-substituted benzenes have zero moment indicates that its six carbon atoms must lie practically in a plane.

The data for a number of derivatives of benzene, with their interpretation in terms of the structures of the several molecules, have been given in a number of places. From these data it has been possible to draw certain conclusions concerning the inclination of various groups to the plane of the benzene ring, and to study the interaction between the groups. But, as far as the author is aware there has not appeared a corresponding systematic study of the electric moments of methane and its derivatives of various sorts although it seems to offer interesting and important information. May I therefore describe to you the results of some work in this direction,[3] done in 1928, but now capable of more exact interpretation because of work done in another field within the year. Methane and ethane are known to be non-polar molecules. If, however, a single hydrogen atom of either is replaced by another atom or group of atoms a polar or dissymmetrical molecule will result. Thus CH_3Cl, C_2H_5OH, CH_3I, C_2H_5Br, CH_3CN, CH_3COOH are polar molecules. Furthermore the amount of dissymmetry is different for the different substituted groups and is practically independent of the length of the hydrocarbon chain. The polarity of the groups increases in the order COOH, I, Cl, OH, CN.

If a second polar atom or group is substituted, either into a methane derivative or on the same carbon atom that was previously substituted in the corresponding ethane derivative it may be expected that there will be a mutual repulsion between the atoms or groups if they carry like charges and a mutual attraction if they are oppositely charged. It is thus possible to explain why dichloromethane has a smaller moment than methyl chloride in spite of the fact that it contains two polar chlorine atoms to one polar atom for the latter, and also why the moment for chloroform, with three polar atoms, is smaller than that of dichloromethane. When the second chlorine atom is substituted there is a mutual repulsion between the negatively charged substituents to cause a deformation of the tetrahedron. The distance from chlorine atom to chlorine atom is now greater than was the distance from the first chlorine to the hydrogen atom replaced in the methyl chloride. The introduction of the third chlorine atom causes a further deformation of the tetrahedron. In other words the distance between any two chlorine atoms has now been further changed. The mutual repulsion between these atoms caused by the introduction of the fourth chlorine atom to form carbon tetrachloride results in a return to the normal tetrahedron. This explanation has more recently been shown to be the probable one by Debye and his collaborators[4] in their studies of X-ray interference maxima with single molecules. These investigators have actually been able to determine the distances between chlorine atoms in these compounds, something which is quite impossible at present with electric moment measurements. If there is substituted an atom or group other than chlorine into the methyl chloride a mutual repulsion will take place if it is another negative group, and an attraction will result if it is a positive group. Thus, measurements made by Fogelberg, working in the laboratory of the author, for the electric moments of the molecules chloronitromethane ($\mu = 2.68 \times 10^{-18}$ e.s.u.) and bromonitromethane ($\mu = 2.56 \times 10^{-18}$ e.s.u.) can be explained only if the angles between the vectors characteristic of the respective substituent groups can be assumed to make an angle of about 140° with each other, rather than 110°, the normal direction angle between valences directed toward the corners of a regular tetrahedron. Both the substituents in each case are negative in character.

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[3] Williams, *Z. phys. Chem.*, A138, 75 (1928).

The statements made above apply equally as well to derivatives of ethane in which two hydrogen atoms attached to the *same* carbon atom are substituted by these simple atoms or groups. If the two atoms or groups are substituted in such a way that one is attached to each carbon atom other interesting cases present themselves. Before discussing them it will be well to recall the information concerning the electrical symmetry of the isomers of 1, 2 dichloroethylene obtained by Errera.[5] For the *cis*-form an electric moment, $\mu = 1.9 \times 10^{-18}$ e.s.u. is found, whereas the *trans*-form shows zero polarity in accordance with the geometrical picture adopted in organic chemistry. These compounds differ from the corresponding ethane derivative in that there is a double bond between the carbon atoms which is supposed to prevent rotation of the polar groups about a line joining the centers of the carbon atoms. For the ethane derivative organic chemists have assumed a "free" rotation about this line joining the centers because there is a single bond between

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[4] Debye, Bewilogua, and Ehrhardt, *Ber. Sächs. Akad. Leipzig*, **81**, 29 (1929); *Physik Z.*, **31**, 419 (1930); Debye, *Z. Elektrochem.*, **36**, 612, 1930.

[5] Errera, *Physik. Z.*, **27**, 769 (1926).

Fig. 1*a*.

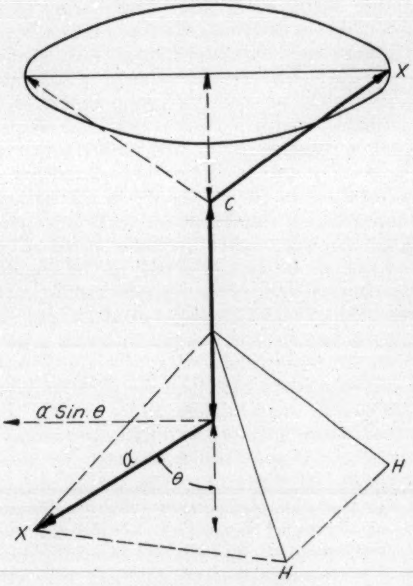
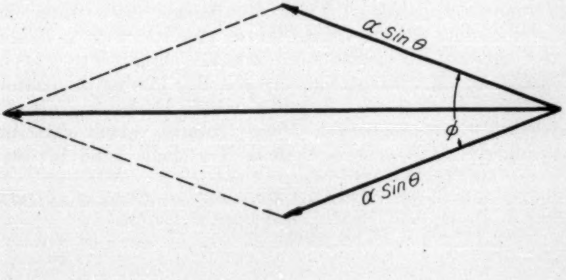

X
C
α Sin θ
H
α
θ
X
H

Fig. 1*b*.


α Sin θ
φ
α Sin θ

them.[6] The electric moment data,[3] and more recently, studies of the scattering of X-rays [7] indicate that the term free rotation cannot be interpreted to mean an unhindered rotation as has been done in a number of cases. This can best be demonstrated with the help of Fig. 1. If it be assumed that there is free and unhindered rotation about the carbon to carbon linkage, then every position of rotation of the vector characterizing the magnitude and direction of the second group is equally probable with respect to a supposedly fixed position of the vector representing the first group. This means, in effect, that there can be no mutual repulsion or attraction between the two groups and the characteristic vectors will each make an angle of 70° (θ in the diagram) with the line joining the centers of the carbon atoms. In the case of two like substituents this angle can be shown to be much less than this value.

Assuming once more that there actually is such a free rotation it was shown [3] for the case of like substituents that

$$\mu = \sqrt{2}\, a \sin \theta,$$

where μ = average electric moment of the molecule obtained directly by experiment,

a = vector value characteristic of the individual substituent atom or group,

θ = angle described above.

This is a perfectly general formula for this kind of molecule and should indicate directly whether or not a free rotation may be assumed, because if the angle θ turns out to be less than 70° it indicates a repulsion (since the case of like groups is being considered) between groups and therefore either a hindered rotation in which certain relative positions of the vectors are favored over others, *or* no rotation at all. The results of the experiments and calculations for five molecules of this type are given in Table 1.

The angles are considerably less than 70° in the case of the first four compounds, which indicates that there must be an appreciable mutual repulsion between the polar groups.

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[6] See, for example, Sugden, "The Parachor and Valency," Routledge, London, 1930.

[7] Debye, *Physik Z.*, **31**, 142 (1930); *Proc. Phys. Soc. London*, **42**, 340 (1930).

TABLE I.

Data for Angle θ for Compounds of Type $\alpha CH_2 . CH_2\alpha$.

Molecule.	a.	μ.	sin θ.	θ°.
Ethylene Chloride	2.0	1.75	0.62	38
Ethylene Bromide	1.86	1.4	0.53	32
Ethylene Iodide	1.65	1.3	0.55	33
Ethylene Glycol	1.64	1.5	0.65	41
Ethylene Cyanide	3.4	3.8	0.80	54

The result for ethylene glycol must be considered to be uncertain for two reasons, as follows:

1. The OH group is being treated as behaving as simply as a chlorine atom, and
2. Because of the very limited solubility of ethylene glycol in benzene the value for its moment, $\mu = 1.5 \times 10^{-18}$ e.s.u. can only be approximately correct.

At any rate it is clear that the angle is much less than that which would indicate normal directions for the substituent atoms. It is interesting to note that ethylene cyanide gives an angle of distinctly larger magnitude. It may be explained by considering this molecule to consist of a four carbon chain, and since the polar groups are now more widely separated there will be a smaller interaction between them.

Debye has concluded from his measurements of the scattering of X-rays by single ethylene chloride molecules that a really free rotation does not exist because the interference maxima caused by the chlorine atoms are quite distinct and indicate a constant distance of separation between them. It appears, therefore, that an equilibrium position, similar to the *trans*-isomer of 1, 2 dichloroethylene, actually exists. It is necessary, therefore, in order to explain the finite dipole moment to assume the existence of thermal oscillations of the two parts of the molecule about their equilibrium positions and around the carbon to carbon bond. If this assumption is correct the electric moment of molecules of this type should increase with increasing temperature because under these conditions a more and more nearly free heat rotation should result. At the very lowest temperatures the molecule should approach a rigid condition corresponding to a trans isomer. Data are not as yet sufficient to determine whether or not these assumptions are correct but they are sufficient to indicate that really free rotations do not exist until the polar parts of the molecules are separated by a distance equivalent to a chain of something like five carbon atoms.

These examples are sufficient to indicate the kind of information concerning the shape or structure of a molecule that can be obtained from a study of the temperature variation of the dielectric constant, or something derived from it, that is, the method of the binary liquid mixture. It is interesting that just the thing which has enabled us to determine the electric moment of a molecule at a single temperature using the binary mixture, namely, the variation of dielectric constant with frequency, also enables us to learn something about the size of molecules and aggregates of molecules. The theory which makes this possible (as well as the theory which makes possible the determination of the electric moment of a molecule) has been given in detail by Debye,[8] so that a complete theoretical treatment will not be given here. It does seem worth while, however, to discuss certain of its implications, and in order to do this it will be necessary to have an outline of the theory in mind.

THE SIZE OF MOLECULES AND PARTICLES.

From what has been said in the previous section it is evident that any polarization due to the orientation of a polar molecule will vanish when the frequency of the applied field becomes sufficiently high and under these conditions there will remain simply a polarization due to deformation of the molecule. The transition from both orientation and distortion effects to the distortion effect alone should occur in the frequency region ν_c which is approximately defined by the equation $\nu_c \tau = 1$, where τ, called the time of relaxation of the polar molecules, is the time required for the molecules to assume a random distribution after the field has been cut off. Thus, liquids which at ordinary frequencies are characterized by unusually high dielectric constants show a marked decrease in dielectric constant when the frequency is passing through this critical region. These liquids are in every case polar in character, and as such contain polar groups, for example, NH_2, I, COOH, CN, etc. Furthermore, the polar molecules are being oriented in a viscous medium so that there will be frictional losses in the frequency region where the dielectric constant is changing, and a dissipation of some of the energy of the field in the form of heat will result.

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[8] Debye, Handbuch d. Radiologie, VI, 597 (1925); "Polar Molecules," Chem. Catalog Co., New York, 1929.

The distribution function of the dipole moments when affected by a field of variable frequency has been obtained by Debye, making use of the general mathematical basis provided by the Einstein theory of the Brownian movement. The generalized distribution law turns out to be

$$f = A\left[1 + \frac{\mu F_0}{kT} e^{-\left(\frac{2kT}{\zeta}\right)t} \cos\theta \right], \tag{7}$$

where f = factor to give the number of molecules which at a given time have their moments in the direction of a given solid angle, thus $fd\Omega$ is the number of molecules whose moments are directed in solid angle $d\Omega$ at the beginning of an interval of observation,

A = constant,

μ = electric moment of the molecule,

F_0 = field strength,

k = Boltzmann constant,

T = absolute temperature,

ζ = constant measuring the inner friction,

t = time,

θ = angle between axis of solid angle $d\Omega$ and a second solid angle $d\Omega'$.

The variable part of this distribution function will be reduced to $1/e$ of its initial value after a time of $\zeta/2kT$ seconds has elapsed. Therefore the time of relaxation of a molecule is defined by the equation

$$\tau = \frac{\zeta}{2kT}.$$

The treatment to this point, while it has involved certain approximations, has, nevertheless, made use only of assumptions which seem to be justified by the vast amount of dielectric constant work which has been done in recent years with vapors and dilute solutions of polar molecules in non-polar solvents. There has already been introduced the assumptions concerning the internal field which have been made by Clausius and Mosotti, previously referred to, but here again difficulty may be avoided by working with dilute solutions of polar molecules dissolved in a non-polar solvent.

There is another source of trouble which seems much more serious. The relaxation time τ in our discussion is as yet an unknown parameter. It has been evaluated by means of a calculation of Stokes who showed that when a sphere of radius a rotates in a liquid of inner friction constant η the frictional torque is $8\pi\eta a^3$ times the angular velocity of the sphere. For such a sphere, then

$$\zeta = 8\pi\eta a^3 \tag{8}$$

and

$$\tau = \frac{4\pi\eta a^3}{kT}. \tag{9}$$

The difficulty contained therein is that for "η" there must be used the inner friction of the liquid or solution as it is ordinarily defined and determined.

Under ordinary experimental conditions where frequencies as high as those corresponding to a wave-length of 10 meters can be obtained for dielectric constant studies with molecules having a radius of 5×10^{-8} cm. in solution in a non-polar solvent it will be necessary that the effective inner friction constant be in the neighborhood of 10 poises in order to observe the dispersion of the dielectric constant. To obtain a medium of such a high viscosity it is necessary to resort to the use of heavy mineral oils, a procedure which must be regarded as unsatisfactory. Little is known concerning the actual structures of the molecular constituents of these oils except that they are irregular in shape. The objection may then be raised that the viscosity of the oil, or of a dilute solution of polar molecules in the oil, measured in a viscosimeter and determined by the rate at which these irregular solvent molecules slip past each other to give a macroscopic viscosity, is hardly the viscosity that one should use in the equation for the time of relaxation. The viscosity needed in that equation should be a "microscopic" viscosity, and should measure the frictional resistance of the solvent molecules to the *rotation* of the polar solute molecules. In the case being discussed the solute molecule will be considerably smaller than these solvent molecules between which they have to rotate or orient. Indeed, there is evidence that the measured macroscopic viscosity of such systems will always be too high so that any calculated molecular radius, making use of the exact equation of Debye, will be too small, since the frequency ω at which the absorption index κ reaches a maximum is the quantity readily calculated from the observed dielectric constant. This frequency of maximum energy loss is given by the formula

$$\omega\tau = \frac{\epsilon_0 + 2}{\epsilon_1 + 2}\sqrt{\frac{\epsilon_1}{\epsilon_0}}, \tag{10}$$

where $\omega = 2\pi\nu$,

ϵ_0 = value of dielectric constant for high frequencies (the optical dielectric constant),

and ϵ_1 = the static dielectric constant, observed for $\omega = 0$.

As far as the author is aware but two papers have been published in which studies of this anomalous dispersion in dilute solutions of polar molecules in a non-polar solvent have been reported. Both articles[9, 10] are based upon measurements made in very viscous oils; in the first one the dielectric constants of the systems have been measured as a function of frequency with the viscosity (temperature) constant, and in the second one the dielectric constants of the systems have been measured as a function of temperature using several frequencies. The method of the first one should be better than that of the second because the exact equations of Debye, applicable only to experiments at constant temperature with variable frequency, can be used only as approximations when the temperature is allowed to vary, because here changes in the density of the solutions, and the corresponding variations in ϵ_0 and ϵ_1, are neglected. In the latter case it is also necessary to know the viscosities of the solutions as a function of the temperature as well as of composition. For such systems these viscosities, measured

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[9] Johnstone and Williams, *Phys. Rev.*, 34, 1483 (1929).

[10] Goldammer and Sack, *Physik Z.*, 31, 224 (1930).

in the ordinary way, in addition to being hard to justify in the formula for the time of relaxation, are very poorly reproducible.

But even in the case of the work of Johnstone and Williams in which the frequency dependence was studied it can be shown that the experimental results justify only the use of the very approximate formula mentioned in the introduction to this section, namely that

$$\tau = \frac{1}{\nu_c} = \frac{4\pi\eta a^3}{kT}.$$

because of this lack of an exact method of determining the proper inner friction coefficient of the liquid. For these experiments the nitrobenzene molecule was chosen as solute because of its high electric moment, $\mu = 3.9 \times 10^{-18}$ e.s.u., and because its shape must much more nearly approach that of a sphere than a molecule of the type of one of the intermediate alcohols. A value 2.4×10^{-8} cm. was reported for its radius, but it was claimed to be only of the right order of magnitude, and it was further stated that the experiments could not be considered to be a verification of the *quantitative* theory of Debye. That the result came out so nearly correct is due to the fact that the error introduced by the use of the high (macroscopic) viscosity values was compensated by the approximations made in simplifying the formula. This can be demonstrated by carrying through a calculation in which the more exact formulæ are applied to the dielectric constant data.

The calculation, with equations used, for an assumed infinitely dilute solution of nitrobenzene in mineral oil, follows:

$$\tau = \frac{\zeta}{2kT},$$

$$\zeta = 8\pi\eta a^3,$$

$$\chi = \frac{\epsilon_1 + 2}{\epsilon_0 + 2}\,\omega\tau = \text{new variable, introduced for convenience},$$

$$\omega = 2\pi\nu,$$

$$\omega\tau = 2\pi\nu\cdot\frac{4\pi\eta a^3}{kT},$$

$$a = 2.4 \times 10^{-8}\ \text{cm.},$$

$$\omega\tau = 2.67 \times 10^{-8}\nu\eta.$$

Further, let

$$\epsilon_1 = 24.1$$

and

$$\epsilon_0 = 2.40.$$

The value $\epsilon_1 = 24.1$ has been obtained by extrapolation of dielectric constant data for dilute solutions of nitrobenzene in benzene.[11] The value $\epsilon_0 = 2.40$ is taken as an average value for non-polar oils.

Then

$$\chi = \frac{\epsilon_1 + 2}{\epsilon_0 + 2}\,\omega\tau = 1.58 \times 10^{-7}\nu\eta.$$

The potential difference between the plates of the measuring condenser is $V \cdot e^{i\omega t}$. The dielectric constant will now have to be separated into a real part ϵ' and an imaginary part ϵ'', according to the equation

$$\epsilon = \epsilon' - i\epsilon''. \qquad (11)$$

Further, it can be shown that

$$\epsilon' = \epsilon_0 + \frac{\epsilon_1 - \epsilon_0}{1 + \chi^2} = 2.40 + \frac{21.7}{1 + 2.5 \times 10^{-14}(\nu\eta)^2}, \qquad (11a)$$

$$\epsilon'' = \frac{\epsilon_1 - \epsilon_0}{1 + \chi^2} \cdot \chi = \frac{21.7 \times 1.58 \times 10^{-7}\nu\eta}{1 + 2.5 \times 10^{-14}(\nu\eta)^2}. \qquad (11b)$$

It is now possible to construct curves showing ϵ' and ϵ'' as a function of $\nu\eta$, or better, as a function of log $\nu\eta$, from these equations by assuming values of $\nu\eta$. The results of the calculations are given in Table 2 and in Fig. 2.

An inspection of the curves shows that the point of inflection in case of the ϵ' curve, and the maximum in the case of the ϵ'' curve are displaced toward the left from what one would expect if the observed macroscopic viscosities were equivalent to the quantity η in the formula for the time of relaxation, because for a dilute solution of nitrobenzene in mineral oil which showed dispersion in the region of wavelength 20 meters the observed viscosity was over 20 poises. According to the $\nu\eta$ product the viscosity value necessary for coincidence of the observed and calculated curves should

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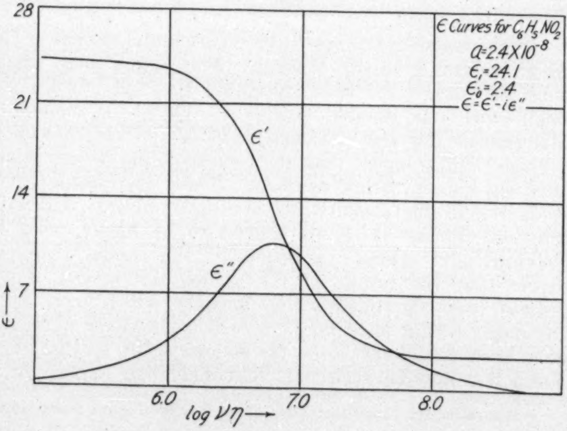
[11] Williams and Schwingel, *Jr. Amer. Chem. Soc.*, **50**, 363 (1928).

TABLE 2.

Values of ε′ and ε″ for Nitrobenzene.

$\nu\eta$.	log $\nu\eta$.	ϵ'.	ϵ''.
0	$-\infty$	24.1	0
4×10^5	5.602	24.0	1.36
$8 \times$	5.904	23.8	2.70
$9 \times$	5.954	23.7	3.03
1×10^6	6.000	23.6	3.34
$2 \times$	6.301	22.2	6.24
$3 \times$	6.478	20.1	8.40
$4 \times$	6.602	17.9	9.80
$5 \times$	6.699	14.8	10.6
$6 \times$	6.778	13.8	10.82
$7 \times$	6.845	12.15	10.8
$8 \times$	6.904	10.75	10.6
$9 \times$	6.954	9.59	10.2
1×10^7	7.000	8.60	9.80
$2 \times$	7.301	4.37	6.23
$3 \times$	7.478	3.32	4.36
$4 \times$	7.602	2.93	3.35
$5 \times$	7.699	2.74	2.72
$6 \times$	7.778	2.64	2.26
$7 \times$	7.845	2.58	1.94
$8 \times$	7.904	2.53	1.71
$9 \times$	7.954	2.51	1.52
1×10^8	8.000	2.49	1.36
∞	$+\infty$	2.40	0

FIG. 2.



be of the order of magnitude 2 poises, in other words the observed true macroscopic viscosities are too high to be used for the inner friction constant of Stokes' law by a factor approaching ten.

If the experiments had been made upon a system of polar molecules dissolved in vaseline or something of still greater consistency there is every reason to believe that the macroscopic viscosities would be too high by a much larger amount. After all little is known concerning the size and, what is more important, the shape of the molecules or aggregates of the heavy oils, cable compounds, and vaselines except that they must be large and irregular. Therefore, if one wishes to test the theory of anomalous dispersion of Debye in a quantitative sense it is not proper to use these solvents of high macroscopic viscosity in order to have the dispersion take place in a region of easily accessible wave-lengths. Rather it will be necessary to work either at very short wave-lengths (of the order of magnitude of centimeters rather than meters) or with very large, nearly spherical molecules, using the more common non-polar substances such as benzene, cyclohexane and carbon tetrachloride for solvent media. If such solvents are not used the effective viscosity acting against the molecular rotation will be quite different from the ordinary value.

If one examines the results of experimental studies of the theory reported in the literature it is found that where molecular radii have been calculated they are always too small, with one notable exception. Thus, the results of Mizushima[11] for the molecular radii of the lower alcohols average slightly higher than the corresponding figures calculated by kinetic theory methods. These experiments were made, not in dilute solution in non-polar solvents, but with the pure liquids which are known to be highly polar in character and always "associated." We should therefore expect that the results of the actual experiments would deviate from the quantitative formulæ which have been calculated for an ideal liquid in which the polar molecules have been considered to be far enough apart so that their interaction may be neglected. Certainly it is impossible to determine the electric moment of a molecule from dielectric constant data for the pure substance in the liquid state or when it is dissolved in another polar solvent, and this case should be quite analogous as far as quantitative results are concerned.

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[11] Mizushima, Sci. Papers Inst. Phys. Chem. Research, Japan, 9, 209 (1928).

As a result of these considerations it appears that the most useful application of the dispersion theory may be in the field of colloid chemistry, for it should make possible the determination of the size of particles suspended in non-polar, or but slightly polar, commercial oil media. These particles will have radii ten to a hundred times as large as those of ordinary molecules and since the radius appears to the third power in the formula for the time of relaxation the region of dispersion should be shifted to easily accessible wave-lengths for measurement. It is of course required that there be a distance of separation of the centers of gravity of the positive and negative electricity in the particles so that they will tend to orient in the field, and for this there will ordinarily be required a certain degree of dissymmetry in shape. It appears, for example, that the way in which to study the rubber substances, particularly the sulphur compounds, is to suspend them in a non-polar medium and study the frequency and temperature variations of the dielectric constant instead of working with hard rubber of the forms which have usually been studied. This statement applies as well to a number of other commercial insulating materials. One of the greatest difficulties will be the actual suspension of these materials in the proper dispersion medium because there is no guiding principle, for the colloid chemistry of the present day teaches us very little concerning the preparation of finely divided systems in any medium other than water.

There is another interesting point which may be mentioned. We have said that there is a real difficulty in the Debye theory because the inner friction constant of the solvent or dispersion medium may be much smaller than its viscosity as ordinarily observed, but it has been suggested only indirectly that the shape of the molecule or particle which is rotating may need to be taken into consideration. The Stokes formula assumes that the rotating particle is spherical in shape. Can we expect a rod or disc shaped molecule or particle to rotate with the same freedom that a spherical one does?

The problem of the heating of dielectrics in rapidly alternating fields has long engaged the attention of many investigators. There has recently been considerable discussion of the above theory of molecular orientation from the standpoint of dielectric losses in insulating materials. It was first brought to the attention of engineers in this country by the excellent articles of Kitchin and Müller[12] and Kitchin,[13] who, if I interpret them correctly, believe that a considerable portion of the dielectric loss in commercial insulating materials is due to the absorption of energy when their constituent dipole molecules are orienting. The following quotation is taken from the synopsis of Kitchin's paper:

"On this theory the anomalous change of dielectric constant and power factor with temperature and frequency is attributed not to impurities or heterogeneity of structure but to molecules containing electric doublets which try to orient themselves in an electric field. The rotation of the dipole molecules in a viscous medium gives rise to a frictional heat loss expressed as power factor, and also to a contribution to the dielectric constant which vanishes when the dipoles are prevented from responding by too great viscosity or too high frequency. The experimental results are discussed on the basis of that theory."

Kitchin and Müller state in their abstract "The existence of anomalous dispersion in the audio and radio frequency region explains the complicated dielectric behavior of many insulating materials and makes it possible to observe an anomalous behavior of the Kerr Effect, or electric double refraction."

Others, as mentioned in the introduction, believe that the so-called dielectric hysteresis is a negligible factor and that if Joule's heat law be properly applied it satisfactorily explains the dielectric losses. Certainly when ions are present, and it is hard to see how they may be avoided in moisture-absorbing dielectrics (even the oils), the major part of the heat developed in these dielectrics can be explained on the basis of an I^2R loss. From calculations which can be made for the heat loss due to dipole orientations in such systems it is difficult to see how this source can account for more than a small percentage of the total loss. Thus Johnstone and Williams [9] calculated the orientation energy of nitrobenzene molecules for a frequency of 10^7 cycles per second and found that the energy dissipation per second per cubic centimeter would be of the order of magnitude 0.01 calorie. Nitrobenzene was chosen for the calculation because of its high dipole moment. If such a dielectric as water containing a very low concentration of ions is placed in a high frequency field much greater power losses result. Hamburger [14] has recently published the results of more exact and detailed calculations and has come to a similar conclusion.

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[12] Kitchin and Müller, *Phys. Rev.*, **32**, 979 (1928).

[13] Kitchin, *Trans. A. I. E. E.*, **48**, 495 (1929).

It is the present opinion of the writer that losses due to dipole orientation may be *sufficient* to account for the observed energy dissipation, in some cases, but this mechanism may not be *necessary* since it appears possible to account for losses of like magnitudes by assuming oscillations of ions. Data are as yet insufficient to decide this point.

THE STRUCTURE OF INSULATING MATERIALS.

In order definitely to decide this and other questions concerning the nature of dielectric loss it seems necessary to learn more about the actual structures of these commercial dielectrics. Whether we deal with liquids or with solids the most useful information that can be obtained will probably come from the applications of methods of colloid chemistry to the study of these substances. In the case of the solid substances like rubber, bakelite, glyptal, wood, and the cellulose esters, the compositions, sizes, and shapes of aggregated molecules themselves are practically unknown. Also, what may be even more important from the standpoint of their insulating qualities, there is little or nothing known about the size, form, and arrangement of the capillaries or voids into which water may be absorbed. These substances in commercial use usually contain electrolyte materials which dissolve in the absorbed water to form conducting solutions which are injurious to the insulating qualities of the material.

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[14] Hamburger, *Phys. Rev.*, **35**, 657 (1930).

It is well known that insulating materials cannot be selected simply according to their moisture contents, thus the insulating properties of gutta percha and rubber change but little under widely differing conditions of moisture while those of wood and textiles might almost be used to indicate changes in the humidity of the air. The synthetic resins and cellulose esters show properties intermediate between these extremes. These differences may perhaps be partially accounted for by differences in the electrolytic impurities present in the various materials, but it seems even more important to have information concerning their actual capillary structures. It may be recalled that Evershed [15] assumed that much of the water contained in insulators is originally in the form of isolated pools, so that the latter could not be responsible for a conductance at the instant of the application of the voltage. These pools were assumed to be electrokinetically spread out into conducting paths in the same manner that water may be forced through a porous diaphragm. But it is hardly necessary, in some cases at least, to have to assume this process of electro-osmosis to be of such fundamental importance, and indeed there are certain objections to so doing. Structural studies of these various types of high molecular (or aggregate) weight polymers indicate fibrous arrangements, and therefore possible conducting paths in a large number of cases, such as cellulose, rubber, the polyoxymethylenes and certain of the glyptal resins. It is certainly common in nature to have fibrous structures, and it is not uncommon to have this structure in synthetic materials.

One of the simplest means of detecting such an arrangement is by means of a study of the swelling of the material in suitable solvents. When a cellulose section swells in water it is not elongated in the direction of the fibers, it expands laterally. Also, if the tensile strength in the longitudinal direction is much greater than that in a direction at right angles to it, it indicates that the primary valence bonds are directed in regular arrangement parallel to the longitudinal direction, and any secondary valence bonds are directed (in general) at right angles to them. The coefficient of thermal expansion should also be much greater in the longitudinal direction if there is such an orderly arrangement of molecules in columns. These structures are also indicated by X-ray investigations. Other substances we believe to have only an amorphous structure.

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[15] Evershed, *Inst. Elec. Eng. Jr.*, **52**, 51 (1914).

If water has been absorbed in the capillary structure of an insulating material and electrolytic substances are present there will be present ions to conduct the current. The ions may be free or they may be adsorbed to the material of the walls. In the small concentrations ordinarily present the ions are more likely to be adsorbed and may even oscillate with an applied alternating potential causing an energy loss. In order to explain a decreasing insulation resistance with increasing voltage the assumption can be made that under these conditions more and more of the adsorbed ions are loosened from the walls and the result is an ionic conduction of the true solution type through the capillaries.

The similarity between the fibrous structure suggested for substances ordinarily thought of as being non-crystalline and the mosaic structure postulated for crystals leads to the interesting possibility that we may be dealing with a process of conduction of like mechanism in both cases. It will be recalled that Smekal[2] believes that the ions which take part in the conduction are concentrated in the positions in the crystal where lattice imperfections are present, the ions moving in either an adsorbed or free condition along paths formed by these fissures. The Smekal point of view, although supported only by indirect evidence, has found favor with a large number of investigators.

A number of physical methods have been applied to the determination of the effective capillary cross-section of not necessarily commercial insulating materials, but substances which are of the same general type. There can be little doubt about the fact that the high molecular (or aggregate) weight organic compounds used as insulators possess colloidal properties and they should be classified as gels or solvated colloids. The methods which have been successful in yielding information concerning the familiar gels of colloid chemistry, the silica and gelatin gels, should be applicable to these substances. One such method may be suggested to illustrate the point. It consists of a study of the rate of diffusion of small molecules either as a gas or from a suitable solution, into or through sections of the materials. The method may be reversed in special cases, thus it should be possible to determine the average pore size in a wood section by observing the rate at which it dries, that is, the rate at which water molecules diffuse out of it.

The fact that the insulating property of various materials is affected in different ways by moisture suggests there is in each case a characteristic arrangement of conducting water paths which depends primarily upon the colloidal structure of the material rather than upon its actual chemical composition. This does not mean that a knowledge of the actual structures of the aggregates themselves is necessarily unimportant, actually this information may indicate the type of capillary structure to be expected.

The author takes this opportunity to thank Mr. J. L. Oncley for material assistance in the preparation of this article.

SUMMARY.

There has been indicated the importance of a knowledge of the chemical and capillary structures of insulating materials if electrical energy losses in them are to be explained. It has been demonstrated that there are available, first, methods by which the structures of their simpler molecules may be determined, second, methods by which the sizes and shapes of their aggregates may be approximated, and third, methods by which it may be expected that something may be learned concerning the size, form and arrangement of their capillaries. In certain cases these methods have been given in some detail, while in others they have only been mentioned. Their nature and the importance of their solution make them deserving of every attention the chemist and physicist can devote to them.

LABORATORY OF COLLOID CHEMISTRY,
UNIVERSITY OF WISCONSIN,
November 1, 1930.

PIEZO-ELECTRIC GAGE AND AMPLIFIER.

BY

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Due to inertia factors in other types of pressure measuring apparatus, the piezo-electric gage has been considered as the solution for overcoming faults due to inertia lag. Yet another problem enters, viz., that of recording accurately the minute quantities of electricity caused by the piezo-electric effect. A number of experimenters have used this effect for pressure measurement. Dr. J. C. Karcher [1] used a ballistic galvanometer but was required to differentiate the curve mechanically to obtain the pressure-time relation. Capt. R. H. Kent, experimenting at the Aberdeen Proving Ground, has used a resistance-coupled amplifier with some success. The aim of the experiments at the Naval Research Laboratory was to improve on the record obtainable by the latter method. This method is the only available one giving directly a pressure-time curve, a very desirable record.

A gage was designed and built using a cartridge of pure bakelite. Within this was placed a pile of twenty-one quartz crystals, each 0.69 inch square by 0.053 inch thick, they being separated by platinum electrodes. Screwed brass ends held them in the cartridge which was then sealed with beeswax. A three-eighths inch piston bore upon this pile. This gage has given very good service and although a few of the crystals cracked, no loss in sensitivity has been noticed. Its maximum load of about 10,000 lb. per sq. in. does not approach the crushing load of the crystal,[2] about 50,000 lb. per sq. in.

With the aid of Dr. L. P. Wheeler of the Laboratory staff in a consulting capacity, a resistance-coupled amplifier of the Western Electric type was built, using two 50-watt power tubes in the third stage.

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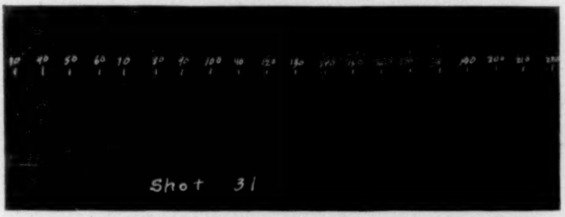
[1] Bureau of Standard Sci. Paper No. 445.

[2] See *Mech. Engineering*, Aug. 1926 (p. 821), Col. W. H. Tschappat.

On the oscillograms it will be noted that timing lines have been placed. These are spaced one thousandth of a second apart. In order to put these on the film, a separate exposure was made and the electrically operated tuning fork that placed them thereon, was stopped while the phenomenon was being recorded, this being done to remove interference of the tuning fork circuit. The electrically operated primer circuit for firing the bomb did not interfere because its pulsations were through when the phenomenon occurred. No serious error is introduced by double exposure since the heavy drum of the film holder changes very little during two successive exposures.

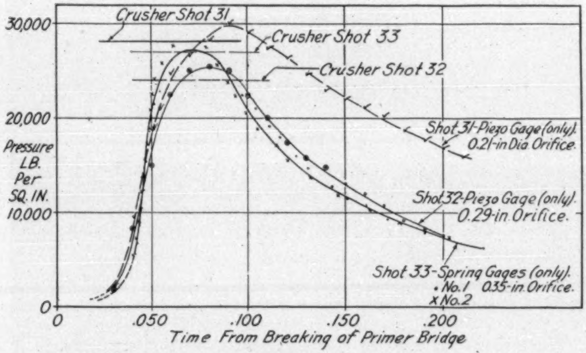
After experimenting with the effects of various interferences, it was found that the shots had to be taken with only the piezo-electric gage registering and with such circuits operating as were absolutely necessary. Hence the amplifier was adjusted under these conditions with the lines from the gage spread about twenty feet apart for most of the distance between the bomb and the laboratory. With 700 volts on the third stage plate circuit and 165 volts on the first two stages good deflections were obtained without apparent distortion.

PLATE I.



Successive records of three rounds were taken that checked very well with similar records of the spring-electric gages. Plate I shows a typical record of the piezo-electric gage as taken under the above conditions. Plate II shows a comparison of curves taken on shots 31, 32, and 33. Shot 33 gave a record on two spring gages, the curve shown being the average. Apparently the piezo-electric gage on shot 32 agrees as well with the average of the two gages on shot 33 as they do between themselves, which condition indicates a reasonable performance of the former. The piezo-electric gage was not calibrated but its maximum deflection was considered as comparable with the reading of the crusher gage on the same shot. Then the other points of the curve were taken proportionately to this maximum point.

PLATE II.



It was found that for the curve of the piezo-electric gage not to be affected appreciably by leakage, insulation resistance had to be, at least, 200 megohms. The final overhead shielded wire between bomb and laboratory gave this in wet weather. With 4 megohms across input grid of amplifier, the effective value of resistance of the input circuit was 3.5 megohms and the capacitance (*C*) 0.016 mf. Without cable attached, *C* became 0.0021 mf.

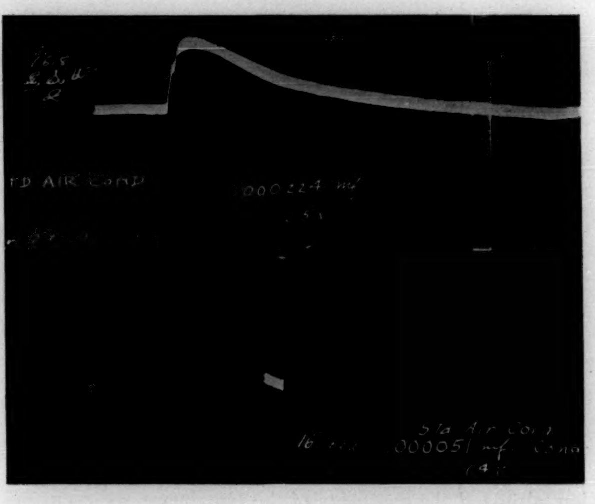
For the study of distortion a series of tests has been accomplished by discharging known quantities of electricity at different voltages into the input side of the amplifier through an approximately known circuit. The inductance of this circuit is small and has generally been considered negligible. The resistance and capacitance are known only approximately and are given above. Nevertheless computations of the discharge curves seem to check the oscillograph record fairly well. One series of condenser discharges in a high resistance circuit has been run on the amplifier, wherein the electric charge (*Q*) was kept constant on the charged condenser and the charging e.m.f. (*E*) and the capacitance (*C*) of the condenser were varied. These were taken in steps as follows:

E (volts).	*C* (mfs.).	*Q* (e.s.u.).
3.8	.001475	16.8
6.0	.000934	16.8
25.0	.000224	16.8
104.0	.000051	16.0

PLATE III.

It is interesting to compare these tests (Plates III and IV). The four curves check very nicely, indicating that as long as *Q* remains constant the amplified curve is the same.

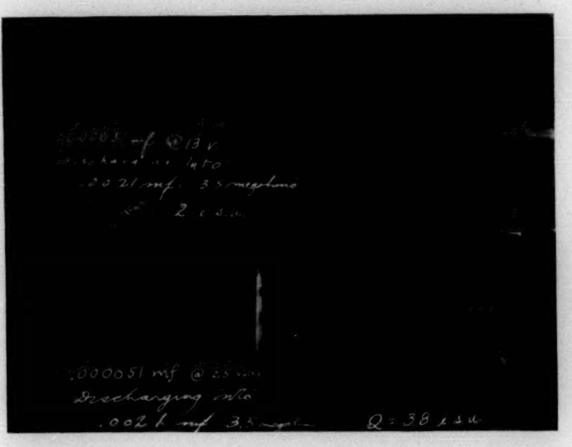
PLATE IV.



The capacitance (C_0) of the circuit into which the condenser was discharged had a value of about .016 mf. which was a great deal higher than that of the discharging condenser. Hence in the majority of these cases the total capacitance ($C_0 + C$) was equal to a great deal more than that of the discharging condenser. Therefore, Q having been constant, the voltage to which the system was raised at time of discharge was about constant, since $eC_t = Q$ and C_t was about equal to C_0, $C_0 + C$ being about identical to C_0. The total capacitance here was $C_t = C_0 + C$.

Thus e_1, e_2, etc., were about equal to each other. This caused the potential across the terminals of the amplifier to have been about constant and, therefore, the output onto the oscillograph should have been constant. According to Plates III and IV this is shown to have been the case. The small jogs on the traces were due to a characteristic of the operating switch. In later work these jogs were eliminated.

Plate V.



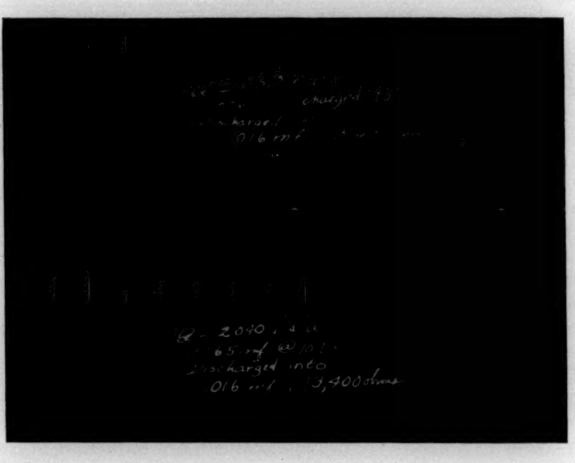
Calculations made from several curves show little distortion, although the constants of the circuit are not accurately known. The following table was made from the lower curve on Plate V and is typical.

(Using Product $RC = 0.0088$.)

t/sec.	log $\epsilon^{t/RC}$.	$\epsilon^{t/RC}$.	Calculated e.	Measured e.
0	0	1	Use $e_0 = 2.76$	2.76
0.001	.0495 096	1.121	2.48	2.45
0.005	.2475 479	1.768	1.56	1.51
0.01	.4950 957	3.127	.88	.89
0.02	.9901 914	9.777	.28	.33
0.03	1.4852 871	30.57	.09	.18

These figures were calculated from the formula $e = e_0\epsilon^{-t/RC}$. The inductance was considered negligible. The lower values do not check, partly on account of difficulty of measuring them from the film; but the upper values seem to give very good checks.

PLATE VI.



Curves on Plate VI are for quantities at 253 and 2040 e.s.u. with the resistance greatly decreased so as effectively to reduce the sensitivity of amplifier. It is difficult to make calculations check with these curves due to lack of knowledge of an accurate value of the inductance, a quantity considered negligible heretofore.

PLATE VII.

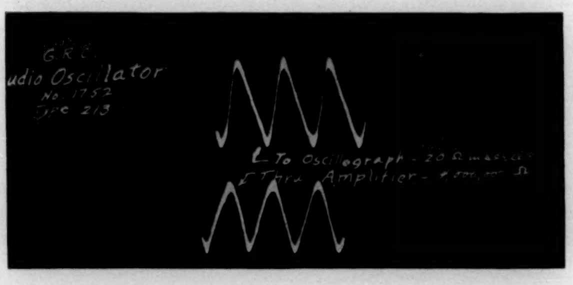


Plate VII shows a record of approximate sine wave off a tuning fork oscillator. The upper wave was taken directly off the fork through a twenty-ohm circuit to the oscillograph vibrator. The lower was from the output of the amplifier and represents the sine wave impressed on a high resistance circuit of about 4,000,000 ohms. It is noted that the latter is more nearly a sine wave, the former having a different slope on one face of a given wave than on the other. These waves not having been taken simultaneously due to lack of power, are not in phase, and hence are not strictly comparable. They do not indicate a perceptible distortion.

Conclusions: It is believed that, although no direct measurements of input and output curves have been taken, no distortion occurs great enough to vitiate results in the measurements of gun powder gas pressures. Care must, however, be taken in adjustment of the amplifier so that there will be a plate output change along the practically straight portion of the grid-voltage plate-current characteristic curve. Then connect the quartz crystal pile so that an increase in pressure on the pile will cause a reduction in the negative bias voltage, tending to make it more positive. This reduces heating in the plate resistors and causes great output in the third stage only while the phenomenon is being measured.

With this adjustment made properly and with the distributed capacitance of the input circuit of such an order that the maximum pressure to be experienced will not cause a blocking of the amplifier, an inch or more amplitude of curve can be obtained, giving an accurately transferable pressure-time curve. This deflection is as much or more than was anticipated when the work was undertaken.

Inductive and leakage interferences are reduced to a negligible amount when well insulated wires are used and their shields fully grounded.

SUMMARY.

The writer built a piezo-electric gage and a resistance-coupled amplifier for recording gun pressures. The gage was tested in a bomb wherein pressures as high as 30,000 lb. per sq. in. were reached.

It is found that a 200-megohm insulation resistance to ground is necessary for proper amplifier response, this resistance having been maintained in wet weather. Good records of as much as an inch amplitude were obtained using standard oscillograph vibrator.

In order to determine the distortion of the amplifier, known quantities of electricity were discharged into a known circuit. These tests indicated that the amplifier responded alike to constant charges impressed at different potentials. Variations from calculated values are less than two and one half per cent.

The Constitution of Osmium and Ruthenium. F. W. ASTON. (*Nature*, Feb. 14, 1931.) These two elements have yielded, though grudgingly, to analysis into isotopes through the use of their volatile tetroxides. "Osmium tetroxide was first experimented with; its action on the discharge, even in the smallest amounts, is quite amazing. The whole mechanism of current flow is violently disorganised and only slowly recovers." It was difficult to obtain spectra of sufficient density. The existence of six isotopes was indicated, of which four were strong and two weak, as follows:

Mass number	186	187	188	189	190	192
Percentage abundance	1.0	0.6	13.5	17.3	25.1	42.6

From these data along with the calculated value of the packing fraction the chemical atomic weight is computed to be 190.31. This makes the accepted value of 190.9 appear too high.

It was almost impossible to get results with ruthenium. The following figures, which are only rough estimates from the photometry of the faint lines, are the best available:

Mass number	96	(98)	99	100	101	102	104
Percentage abundance	5	?	12	14	22	30	17

The atomic weight is calculated to be 101.1. The divergence of this value from the accepted value of 101.7 is in part accounted for by the enhancement of certain lines by mercury. Three of the ruthenium lines have the same mass number as molybdenum lines. The 96 constituent is isobaric not only with a molybdenum line but also with a doubtful isotope of zirconium. Should this be eventually placed beyond question we should have the first instance of an isobaric triplet.

G. F. S.

A THERMIONIC MEGGER WITH LINEAR SCALE.

BY

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The "Radiotron" or three element vacuum tube using "secondary electron emission" to produce a negative resistance was introduced into the radio literature as a "Dynatron" by A. W. Hull[1] and, as its name indicates, is a generator of electrical power and feeds energy into any circuit to which it is connected. It was subsequently developed by the same author and his associates as a receiver for continuous and modulated waves.

Since that time little attention has been paid to this form of vacuum tube until quite recently W. H. Newbold[2] made some suggestions for the use of this "forgotten device" as a self modulated oscillator. Among the various commercial forms of tubes examined by him he especially recommends the R.C.A., UY-224 type because of its marked dynatron characteristics.

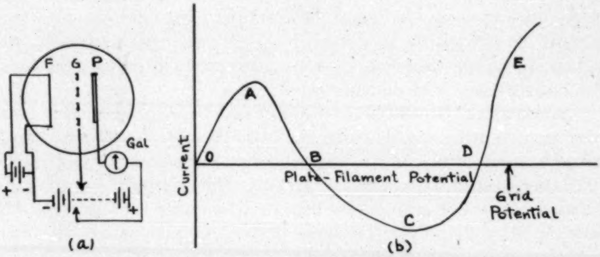
The writer proposes to use any tube with dynatron characteristics, as for instance the "Radiotron," and to calibrate the slope of its characteristic in terms of an external resistance introduced into the plate circuit. If the change in slope of a given phase of the characteristic can be shown to change in proportion to the change in external resistance it should follow that for a predetermined current a voltmeter recording the potential across filament and plate can then be calibrated to read resistances directly. The voltmeter's linear scale is then a direct reading megohmeter.

THE DYNATRON PRINCIPLE.

Three "elements" are arranged as shown in Fig. 1 (*a*), where *F* is an electron emitting hot filament, *G* a grid and *P* a solid plate situated near enough to the grid so that some of the electrons set in motion by the positive grid potential will pass through the holes in the grid and reach the plate. The positive potential at which the grid is maintained is not varied. Changes in its potential only shift the characteristic along its voltage axis.

If a galvanometer is placed in the plate circuit and a variable positive potential, with respect to the filament, is applied to the plate a characteristic curve such as the one shown in Fig. 1 (*b*) is obtained. Starting with the plate *P* at a lower positive potential than the grid *G*, only such electrons leaving the hot filament *F* and accelerated by the grid's potential can reach *P*, as possess enough energy on passing through the holes of the grid to rise against the small retarding potential between *P* and *G*. With increased positive potential of *P*, more and more filament electrons reach *P*, until a maximum current is attained as indicated by the point *A*. This increase in current with increase in potential shows that the plate-filament circuit possesses a positive resistance characteristic as indicated by the positive character of the slope of the curve from *O* to *A*. As the plate voltage is increased beyond the potential of the point *A* the tube begins to exhibit negative resistance properties. This change is usually attributed to a "secondary electron effect" having its origin at the plate. Without going into the controversial aspect of the subject as to just what causes contribute to a decrease in the current between plate and grid, suffice it to say that at certain critical energy values of the impacting

FIG. 1.



(*a*) Shows the three element vacuum tube consisting of filament, grid and plate. The grid is maintained at a constant potential and the plate at a variable potential. The current in the galvanometer under these conditions develops the characteristic curve (*b*).

electrons, reflection, re-emission and inverse photoelectric effects must be taken into consideration in addition to the change in the potential gradient between grid and plate, the sum total of which gradually reduces the electron stream flowing from grid to plate and thence through the galvanometer circuit. This inversion of current continues until the negative current predominates and approaches its saturation value as indicated by the point *C* on the curve.

Between *A* and *C* the tube is developing its negative resistance characteristic associated with electron tubes operating on the "Dynatron" principle. One of the chief reasons that this principle has not been put into greater commercial use, has probably been the inability of the manufacturer effectively to control the re-emission properties of the metal surfaces.

Subsequent increases in potential may carry the plate potential beyond the grid potential values. Under these circumstances the third phase of the characteristic is developed, namely C.D.E. The tube, behaving somewhat analogously to the phase *OA*, develops a characteristic having a positive slope, here referred to as its second positive phase.

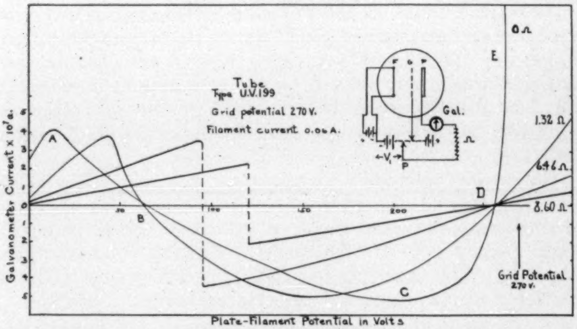
THE SECOND POSITIVE PHASE OF THE CHARACTERISTIC.

The second positive phase of the characteristic of the three element tube to be considered is that portion of the curve extending from *C* through *D* to its saturation value at *E*. It is this phase that is to be used and must be examined in detail.

The change in shape of the characteristic brought about by increasing the grid potential is in general to lengthen the range of the negative resistance of the curve, which in turn pushes the zero-current crossing at *D* to higher potentials. Any three or four element tube possessing the above properties can be used to produce the following results.

A typical characteristic such as is developed by the three element tube UV-199, is shown by the curve *ABCDE* in Fig. 2. The grid potential was 270 volts with a filament current of 0.06 ampere. The galvanometer connected in the plate filament circuit had a 57 megohm sensitivity and an internal resistance of 130 ohms. With a six volt battery supplying the current for the filament and a high external resistance in the plate galvanometer circuit (see insert of Fig. 2), it is possible to distort the original characteristic *ABCDE*. The effect of the external resistance on the stability of the tube is noticed in the peculiar changes accompanying the changes in slope of the negative resistance. When this slope is infinite, the current is unstable and oscillates between its maximum positive and negative values. In this region a small varying potential applied between grid and plate allows the tube to oscillate between these current limits. This instability does not make itself felt in the second positive phase of the characteristic given by *DE*. In this region the slope of the curve is dependent on the magnitude of the external resistance Ω inserted in the plate filament circuit.

FIG. 2.

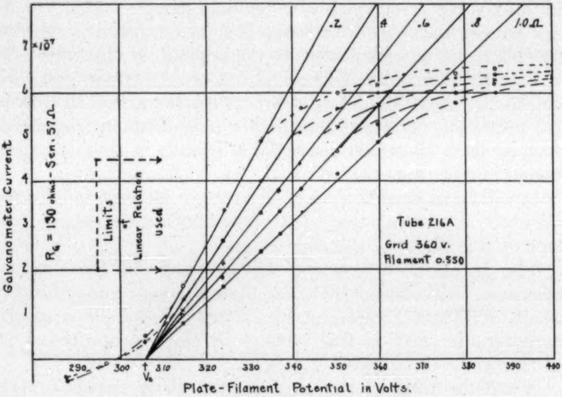


The three element tube shown in Fig. 1 (*a*) with the addition of a resistance in series with the galvanometer. Curves show the change in the characteristic as Ω, the resistance, is changed from zero to ten megohms. *DE* is here referred to as the second positive phase of the characteristic.

ANALYSIS OF THE SECOND POSITIVE PHASE OF THE CHARACTERISTIC

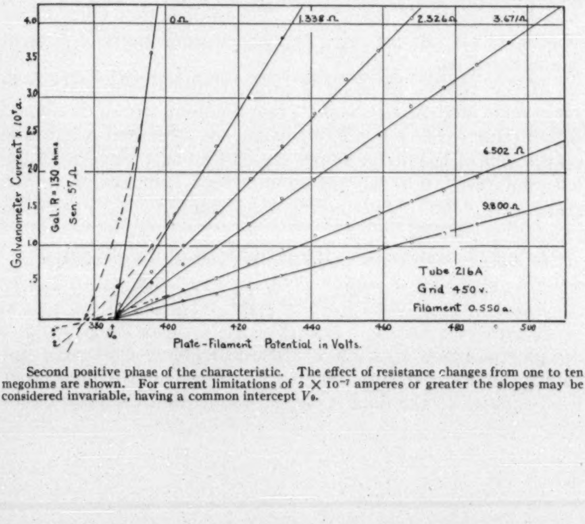
For all values of slope, due to changes in external resistance, the curves pass through a common zero-current point *D* to eventually saturate at very much higher potentials. Detailed structural changes in this phase are shown in Figs. 3 and 4.

FIG. 3.



Second positive phase of the characteristic, referred to in Fig. 2 (*a*) *DE*, is here shown in detail. The resistance changes considered are from .2 to 1.0 Ω in .2 Ω stages. Between current limits indicated, the slopes show no change and have a common intercept V_0.

FIG. 4.



Second positive phase of the characteristic. The effect of resistance changes from one to ten megohms are shown. For current limitations of 2×10^{-7} amperes or greater the slopes may be considered invariable, having a common intercept V_0.

These two figures also show how an increase in grid potential moves the intercept on the voltage axis to higher values.

Fig. 3 is introduced to illustrate how a resistance, varying from 0.1 to 1.0 megohm, changes the slope of the characteristic. In Fig. 4 a resistance changing from one to ten megohms is used to illustrate this. In the latter case, because of the higher grid potential, corresponding higher plate-filament potentials must be used to get an appreciable change in slope per unit change in resistance.

It will be noticed that in both cases an increase in external resistance is accompanied by a corresponding decrease in the slope of this phase of the characteristic.

The question then arises, will a change in the external resistance, introduced in the plate-filament-galvanometer circuit, result in a reproducible change in this slope of the characteristic, and is this change in slope proportional to the change in resistance?

It will be noticed that the characteristic curves of this second positive phase as shown in Figs. 3 and 4 are distinctly linear between currents, ranging from 2 to 4 × 10^{-7} amperes. Using this current range as determining the slopes of the curves it will be observed that all the tangents defining the slopes of the curves have a common voltage intercept for any predetermined grid potential. This implies that Ohm's Law is strictly obeyed provided the above limitations of current are adhered to.

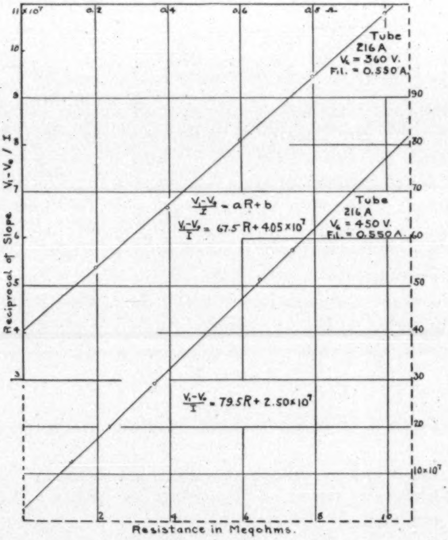
If the change in slope is proportional to the change in resistance introduced in the plate-filament circuit it should follow that a linear relation must be obtained when the reciprocals of the above slopes are plotted as a function of the external resistance, having an intercept comparable to the resistance of the circuit less that resistance referred to as the external resistance.

In Fig. 5 upper curve, the reciprocals of the slopes,

$$\frac{1}{s} = \frac{V_1 - V_0,}{I}$$

are plotted against various standard megohm resistances and shows the linear results sought for. Applying the method of least squares to the data it can be shown that the most probable relations governing these two variables is given by $(V_1 - V_0)/I = 67.9R + 4.03 \times 10^7$ for a grid potential of 360 volts with resistances varying from 0.1 to 1.0 megohm and $(V_1 - V_0)/I = 79.5R + 2.50 \times 10^7$ for a grid potential of 450 volts and when resistances of from one to ten megohms are used. These two typical cases are cited to illustrate the linear relations sought.

FIG. 5.



Shows that the resistance in megohms introduced in the plate-galvanometer circuit is a linear function of the potential applied to the plate. Upper curve for a range of resistance from zero to one megohm. Lower curve for a range of resistance from one to ten megohms.

Hence in general if V_1 is the potential of the point P with respect to the negative end of the filament (see insert, Fig. 2) as read by a voltmeter when a given standard resistance R_s is introduced in the plate-filament circuit, resulting in a deflection I of the current meter G, then our data shows that

$$\frac{V_1 - V_0}{I} = aR_s + b.$$

If then V_2 is the subsequent potential that will cause the current meter to show an identical deflection I for a resistance R_x then

$$\frac{V_2 - V_0}{I} = aR_x + b.$$

Hence

$$\frac{V_1 - V_0 - bI}{V_2 - V_0 - bI} = \frac{R_s}{R_x},$$

or

$$\frac{V_1 - K}{V_2 - K} = \frac{R_s}{R_x}.$$

It then follows that for a known calibration constant, K, determined by the choice of tube, grid potential, filament current and V_0, the voltmeter reading minus a calibration constant K, is proportional to the external resistance under investigation. Hence a voltmeter possessing a linear scale can be converted into a direct reading linear megohm scale, differing from it only by a zero shift in the scale amounting to an experimentally determinable value K.

To illustrate the relative magnitude of the constants we must again refer to Fig. 4. Here

$$\frac{V_1 - V_0}{I} = 79.5R + 2.50 \times 10^7,$$

for a converted voltmeter scale reading from zero to 10 megohms.

Then for a galvanometer reading of 4.00×10^{-7} amperes and a calibration value of V_0 equal to 385.0 volts and a standard megohm in the circuit,

$$\frac{V_1 - 385.0}{4.00 \times 10^{-7}} = 79.5 \times 10^6 + 2.50 \times 10^7.$$

Hence

$$V_1 - 385.0 = 31.80\Omega_1 + 10.00.$$

If now the standard megohm is replaced by an unknown resistance R_x and a potential V_2 applied to reproduce in the current meter a reading of 4.00×10^{-7} amperes it follows that

$$V_2 - 385.0 = 31.80\Omega_x + 10.00.$$

Hence the ratio of the unknown resistance Ω_x measured in megohms is to the known standard one megohm resistance Ω_1 as

$$\frac{V_1 - 395.0}{V_2 - 395.0} = \frac{\Omega_1}{\Omega_x},$$

so that if Ω_1 is one megohm and V_1 the potential in volts necessary to produce the required constant galvanometer deflection, then in general,

$$V_2 - k = c\Omega_x.$$

Hence a linear voltmeter scale can be calibrated to represent a linear megohm scale.

I take pleasure in acknowledging the assistance of my former students, Karl Morgan and H. B. Mulkey, in taking the many painstaking observations.

CHAPEL HILL, N. C.
January, 1930.

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3. W. H. NEWBOLD. *Q. S. T.* (Feb., 1930).

Method of Measuring Strains between Glazes and Ceramic Bodies. H. G. SCHURECHT AND G. R. POLE. (*Bureau of Standards J. of Res.*, Vol. 5, No. 1.) The state of compression or of tension in the glaze or other finish upon the surface of ceramic bodies determines subsequent crazing or shivering. "If a glaze is in strong compression on newly fired bodies it is not likely to be thrown into tension from the subsequent expansion of the body. It is therefore less apt to develop delayed crazing than a glaze in tension or only under slight compression. If a ceramic finish is under excessive compression it may shiver." The purpose of this investigation is to present a method of measuring by simple and rapid means the strains existing between finishes and the bodies they cover.

Hollow cylindrical rings two inches in diameter with the glaze on the outside were used. After firing cuts or holes were made in one edge of the ring and into these were inserted thin capillary tubes to act as markers. The distance between them was then measured by a micrometer microscope. The ring was cut open between the two reference marks and the distance between them was again measured. If the glazes were stretched the ring expands after being cut; if in compression the marks are closer together. The same set of measurements was made also with rings without glaze. Thus the effect of the finish was found and the state whether of tension or of compression was deduced.

One conclusion drawn is this "It seems, therefore, that the use of glazes which are under strong compression on newly fired ware would tend to obviate delayed crazing due to moisture."

G. F. S.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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OSCILLATIONS OF A LINKAGE MECHANISM.

We will consider a mechanism similar to a governor linkage which is rotated about a vertical shaft with angular velocity $\dot{\phi}$. The linkage consists of four equal bars forming a rhombus, each of length $2a$ and of mass m and uniformly distributed. The lower vertical joint is fixed to the shaft, while the upper vertical joint can slide freely along the shaft. The horizontal joints are connected by a spring [1] with modulus of elasticity μ. The initial unstrained length of spring is $2c$. The angle of any rod with respect to the vertical is θ, that is the total angle at the bottom or upper joints between the rods is 2θ.

The coördinates of the system are evidently θ and ϕ. The kinetic energy can be shown to be,

$$T = ma^2[8(\sin^2\theta + \tfrac{1}{3})\dot{\theta}^2 + \tfrac{8}{3}\sin^2\theta\cdot\dot{\phi}^2]$$

and the potential energy due to gravity and the spring is,

$$V = 8mga\cos\theta + 2\mu\int_c^{2a\sin\theta}\frac{y-c}{c}\cdot dy$$

$$= 8mga\cos\theta + \frac{\mu}{c}(2a\sin\theta - c)^2.$$

The momentum corresponding to the coördinate θ is

$$\frac{\partial T}{\partial\dot{\theta}} = 2ma^2[8(\sin^2\theta + \tfrac{1}{3})\dot{\theta}] = p_\theta$$

---

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

[1] Constructively, two parallel springs on either side of shaft could be used, or simply a fixed guide shaft at the top, the rotating shaft terminating at lower hinge.

Concluded from p. 505, vol. 211, April, 1931.

and the momentum corresponding to the coördinate ϕ is

$$\frac{\partial T}{\partial \phi} = \frac{16}{3} ma^2 \sin^2 \theta \phi = p_\phi,$$

which is obviously also equal to the angular momentum, A.M., about the shaft axis.

The equations of motion are,

$$16ma^2[(\sin^2 \theta + \tfrac{1}{3})\ddot{\theta} + \sin \theta \cdot \cos \theta \cdot \dot{\theta}^2] - \frac{16}{3} ma^2 \sin \theta \cos \theta \dot{\phi}^2$$
$$= 8mga \sin \theta - \frac{4a\mu}{c}(2a \sin \theta - c) \cos \theta$$

and

$$\frac{16}{3} ma^2[\sin^2 \theta \cdot \ddot{\phi} + 2 \sin \theta \cos \theta \dot{\theta} \dot{\phi}] = \Phi.$$

The latter equation is obviously the rate of change of A.M. about the shaft axis equal to the shaft torque Φ.

If we assume an oscillation about the kinetic equilibrium position, let ω = the angular velocity corresponding to this position, and θ_0 the corresponding angle of the rods with respect to the vertical. Then,

$$\theta = \theta_0 + \epsilon, \qquad \dot{\theta} = \dot{\epsilon}, \qquad \phi = \omega + \dot{\eta}.$$

The equilibrium position is given by the expression,

$$\frac{4a\mu}{c}(2a \sin \theta_0 - c) \cos \theta_0 = 8mga \sin \theta_0 + \frac{16}{3} ma^2 \sin \theta_0 \cos \theta_0 \cdot \omega^2.$$

Neglecting the displacements and velocities from the equilibrium position of higher order the equations of oscillation are,

$$16ma^2\left(\sin^2 \theta_0 + \frac{1}{3}\right)\ddot{\epsilon} + \left[\left(\frac{4a\mu}{c}(2a \cos 2\theta_0 + c \sin \theta_0) - 8mga \cos \theta_0\right) - \frac{16}{3} ma^2 \cos 2\theta_0 \cdot \omega^2\right]\epsilon$$
$$- \frac{16}{3} ma^2 \sin 2\theta_0 \omega \dot{\eta} = 0, \qquad (1)$$

$$\frac{16}{3} ma^2[\sin^2 \theta_0 \cdot \ddot{\eta} + 2 \sin \theta_0 \cos \theta_0 \cdot \omega \dot{\epsilon}] = \Phi. \qquad (2)$$

These equations may be written in terms of the coefficients, P, Q, K, R, and M and N. If we suppose an impressed harmonic torque of some multiple of the shaft frequency, a very possible condition in apparatus of this type, we have,

$$\begin{aligned} P\ddot{\epsilon} + (Q - K\omega^2)\epsilon + R\omega\dot{\eta} &= 0, \\ M\ddot{\eta} + N\omega\dot{\epsilon} &= \Phi_0 \cos m\omega t. \end{aligned}$$

If we let, $\epsilon = A \sin m\omega t$, $\eta = B \cos m\omega t$, then,

$$A = \frac{-\,m\omega^2 R\Phi_0}{D}, \qquad B = \frac{-\,(Q - (m^2P + K)\omega^2)}{D}$$

and

$$D = \begin{vmatrix} m\omega^2 N, & -\,m^2\omega^2 M \\ -\,(m^2P + K)\omega^2 + Q, & -\,m\omega^2 R \end{vmatrix}.$$

If the shaft torque Φ is due to the reaction of a torsional spring coupling, then $\Phi = -\,k(\phi - \omega t) = -\,k\eta$ and the critical speeds are determined from the determinate,

$$\begin{vmatrix} m\omega^2 N, & (k - m^2\omega^2)M \\ Q - (m^2P + K)\omega^2, & -\,m\omega^2 R \end{vmatrix} = 0.$$

Finally, if the apparatus is connected through an elastic shaft to a mass of moment of interia I an additional degree of freedom ψ is introduced, then

$$\Phi = -\,k(\phi - \psi) = -\,k[(\phi - \omega t) - (\psi - \omega t)] = -\,k(\eta - q)$$

and the equations of oscillation become

$$\begin{aligned} P\ddot{\epsilon} + (Q - K\omega^2)\epsilon + R\omega\dot{\eta} &= 0, \\ M\ddot{\eta} + N\omega\dot{\epsilon} &= -\,k(\eta - q), \\ I\ddot{q} &= k(\eta - q) + \Phi'. \end{aligned}$$

THE DYNAMICAL EQUATIONS FOR A SYSTEM OF VIBRATING DISKS.

We will neglect the gyroscopic forces. The polar moment of inertia through the c. of g. is mk^2. Assume the c. of g. offset from the center of shaft by h. Let ϕ be the torsion angle, i.e., the angle turned by vector h in the plane of the disk with respect to the y axis.

Then for a single disk if y_g and z_g are the coördinates of the c. of g., y_1 and z_1 the coördinates of the center of the disk, the kinetic energy is,

$$T = \tfrac{1}{2}m(k^2\dot{\phi}^2 + \dot{y}_g^2 + \dot{z}_g^2)$$

or, alternately,

$$T = \tfrac{1}{2}m((k^2 + h^2)\dot{\phi}^2 + \dot{y}_1^2 + \dot{z}_1^2 + 2h\phi(\dot{z}_1 \cos \phi - \dot{y}_1 \sin \phi)).$$

The work of the reactive forces of the shaft and weight of the disk, are

$$-\delta V = -Y\delta y_1 - Z\delta z_1 - mg\delta z_g + M\delta\phi$$

where

$$\begin{aligned} y_1 &= y_g - h \cos \phi, & z_1 &= z_g - h \sin \phi, \\ \delta y_1 &= \delta y_g + h \sin \phi \cdot \delta\phi, & \delta z_1 &= \delta z_g - h \cos \phi \cdot \delta\phi. \end{aligned}$$

The equations of motion are, therefore, with coördinates $(y_g z_g \phi)$

$$\left| \begin{aligned} m\ddot{y}_g &= -Y, \quad m\ddot{z}_g = -Z - mg \\ mk^2\ddot{\phi} &= M - Yh \sin \phi + Zh \cos \phi \end{aligned} \right| \text{(I)}$$

or alternately, with coördinates $(y_1 z_1 \phi)$

$$\left| \begin{aligned} & m(\ddot{y}_1 - h \sin \phi \cdot \ddot{\phi} - h\dot{\phi}^2 \cos \phi) = -Y \\ & m(\ddot{z}_1 + h \cos \phi \cdot \ddot{\phi} - h\dot{\phi}^2 \sin \phi) = -Z - mg \\ & m[(k^2 + h^2)\ddot{\phi} + h(\ddot{z}_1 \cos \phi - \ddot{y}_1 \sin \phi) = M - mgh \cos \phi \end{aligned} \right| \text{(II)}$$

The system II is expressed in terms of the coördinates of the shaft. In system I, the moment equation can be simplified for a single disk, since $-Y_1 = -Cy_1 = -C(y_g - h \cos \phi)$ and $-Z_1 = -Cz_1 = -C(z_g - h \sin \phi)$, so that

$$mk^2\ddot{\phi} + m(\ddot{z}_g Y_g - \ddot{y}_g Z_g) = M - mg \cdot y_g,$$

which is the equation of angular momentum about the shaft bearing axis.

The equations II show that the lateral and torsional vibrations are not independent, except for the special case when the eccentricity of the disk h is *nil*.

If we neglect the weight of the disk, noting $\phi = \omega t + \epsilon$ where ω = the mean angular velocity of the shaft, and assuming $\cos(\omega t + \epsilon) = \cos \omega t$ (approx.), we have for the loadings on the shaft,

$$\begin{aligned} Y &= -\, m(\ddot{y}_1 - \ddot{\epsilon} h \sin \omega t - h(\omega^2 + 2\omega\dot{\epsilon}) \cos \omega t), \\ Z &= -\, m(\ddot{z}_1 + \ddot{\epsilon} h \cos \omega t - h(\omega^2 + 2\omega\dot{\epsilon}) \sin \omega t), \\ -\, M &= -\, m(k^2 + h^2)\ddot{\epsilon} - mh(\ddot{z}_1 \cos \omega t - \ddot{y}_1 \sin \omega t). \end{aligned}$$

These equations offer an interesting verification. If we assume rotating axis y^1 and z^1, along, and normal to, h with moving origin having coördinates $(y_1 z_1)$ evidently, the acceleration along y^1 or h for the c. of g. is

$$\ddot{y}_1 \cos \omega t + \ddot{z}_1 \sin \omega t - h\omega^2 - 2\omega h \dot{\epsilon}$$

and the acceleration normal to h is

$$\ddot{z}_1 \cos \omega t - \ddot{y}_1 \sin \omega t + h\ddot{\epsilon}.$$

Resolving along the fixed axis y and z the acceleration components in the above equations are obtained. Evidently the moment equation is taken about the moving origin $(y_1 z_1)$ where the reaction components $-\,Y$ and $-\,Z$ of the shaft have no moments.

For a single loaded shaft, the potential energy is, due to bending,

$$V = \tfrac{1}{2} a_{11}(y_1^2 + z_1^2) = \tfrac{1}{2} b_{11}(Y^2 + Z^2),$$

then

$$Y = a_{11} y_1, \qquad Z = a_{11} z_1$$

where

$$a_{11} = \frac{1}{b_{11}},$$

$$y_1 = \frac{\partial V}{\partial Y} = b_{11} Y, \qquad z_1 = b_{11} Z.$$

If we assume a torsion moment, $M = -\, c(\omega t - \phi) = -\, c\epsilon$ the equations become

$$\begin{aligned} m\ddot{y}_1 + a_{11} y_1 &= mh(\omega^2 + 2\omega\dot{\epsilon}) \cos \omega t + m\ddot{\epsilon} h \sin \omega t, \\ m\ddot{z}_1 + a_{11} z_1 &= mh(\omega^2 + 2\omega\dot{\epsilon}) \sin \omega t - mh\ddot{\epsilon} \cos \omega t, \\ m(k^2 + h^2)\ddot{\epsilon} + c\epsilon &= \underset{\rightarrow}{mh} \cdot (\ddot{y}_1 \sin \omega t - \ddot{z}_1 \cos \omega t). \end{aligned}$$

For a system of vibrating disks attached to a multibearing shaft, the kinetic energy in terms of the coördinates of the shaft, is

$$T = \tfrac{1}{2} \sum_{1}^{m} m_m[(k_m^2 + h_m^2)\phi_m^2 + \dot{y}_m^2 + \dot{z}_m^2 + 2h\phi_m \times (\dot{z}_m \cos \phi_m - \dot{y}_m \sin \phi_m),$$

and the potential energy has the form

$$\begin{aligned} V = \tfrac{1}{2}[&a_{11}(y_1^2 + z_1^2) + a_{22}(y_2^2 + z_2^2) + a_{33}(y_3^2 + z_3^2) \\ &+ 2a_{12}(y_1y_2 + z_1z_2) + 2a_{23}(y_2y_3 + z_2z_3) \\ &+ 2a_{31}(y_3y_1 + z_3z_1) + \cdots] \\ &+ \tfrac{1}{2}[c_{12}(\phi_2 - \phi_1)^2 + c_{23}\cdot(\phi_3 - \phi_2)^2 \cdots]. \end{aligned}$$

In general, applied torques act on various disks of the nature $\Phi = M \sin (\eta\omega t + \alpha)$. On substituting $\phi = \omega t + \epsilon + \phi_0$ we have finally the system of equations,

$$\begin{aligned} &m_1\ddot{y}_1 + a_{11}y_1 + a_{12}y_2 \cdots \\ &\quad = m_1h_1(\omega^2 + 2\omega\dot{\epsilon}_1) \cos (\omega t + \phi_{01}) + m_1\ddot{\epsilon}_1h_1 \sin (\omega t + \phi_{01}), \\ &m_2\ddot{y}_2 + a_{21}y_1 + a_{22}y_2 \cdots \\ &\quad = m_2h_2(\omega^2 + 2\omega\dot{\epsilon}_2) \cos (\omega t + \phi_{02}) + m_2\ddot{\epsilon}_2h_2 \sin (\omega t + \phi_{02}), \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &m_1\ddot{z}_1 + a_{11}z_1 + a_{12}z_2 \cdots \\ &\quad = m_1h_1(\omega^2 + 2\omega\dot{\epsilon}_1) \sin (\omega t + \phi_{01}) - m_1\ddot{\epsilon}_1h_1 \cos (\omega t + \phi_{01}), \\ &m_2\ddot{z}_2 + a_{21}z_1 + a_{22}z_2 \cdots \\ &\quad = m_2h_2(\omega^2 + 2\omega\dot{\epsilon}_2) \sin (\omega t + \phi_{02}) - m_2\ddot{\epsilon}_2h_2 \cos (\omega t + \phi_{02}), \\ &m_1(k_1^2 + h_1^2)\ddot{\epsilon}_1 + c_{12}(\epsilon_1 - \epsilon_2) \\ &\quad = m_1h_1(\ddot{y}_1 \sin (\omega t + \phi_{01}) - \ddot{z}_1 \cos (\omega t + \phi_{01})) + \Phi_1, \\ &m_2(k_2^2 + h_2^2)\ddot{\epsilon}_2 + c_{12}(\epsilon_2 - \epsilon_1) + c_{23}(\epsilon_2 - \epsilon_3) \\ &\quad = m_2h_2(\ddot{y}_2 \sin (\omega t + \phi_{02}) - \ddot{z}_2 \cos (\omega t + \phi_{02}) + \Phi_2. \end{aligned}$$

For small eccentricities, h, the lateral and torsional vibrations become independent and the equations reduce to the approximate form.

$$\begin{aligned} m_1\ddot{y}_1 + a_{11}y_1 + a_{12}y_2 \cdots &= m_1h_1\omega^2 \cos (\omega t + \phi_{01}), \\ m_2\ddot{y}_2 + a_{21}y_1 + a_{22}y_2 \cdots &= m_2h_2\omega^2 \cos (\omega t + \phi_{02}), \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ m_1\ddot{z}_1 + a_{11}z_1 + a_{12}z_2 \cdots &= m_1h_1\omega^2 \sin (\omega t + \phi_{01}), \end{aligned}$$

$$m_2\ddot{z}_2 + a_{21}z_1 + a_{22}z_2 \cdots = m_2h_2\omega^2 \sin(\omega t + \phi_{02}),$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$m_1k_1^2\ddot{\epsilon}_1 + c_{12}(\epsilon_1 - \epsilon_2) = \Phi_1,$$
$$m_2k_2^2\ddot{\epsilon}_2 + c_{12}(\epsilon_2 - \epsilon_1) + c_{23}(\epsilon_2 - \epsilon_3) = \Phi_2, \text{ etc.}$$

The condition for the whirling speeds is when the determinate

$$\begin{vmatrix} a_{11} - m_1\omega^2, & a_{12}, & \cdots \\ a_{21}, & a_{22} - m_2\omega^2, & \cdots \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \end{vmatrix} = 0$$

for lateral vibrations,

$$\begin{vmatrix} c_{12} - m_1k_1^2\omega^2, & c_{12}, & \cdots \\ -\,c_{12}, & c_{12} + c_{23} + m_2k_2^2\omega^2, & \cdots \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \end{vmatrix} = 0$$

for torsional oscillations.

Alternately, we may express the potential in terms of the applied forces and redundant reactions as for indeterminate bearing reactions. Since that part of the potential due to torsion is independent of the lateral loadings, we will maintain it in its previous form. We finally arrive at the lateral displacements as a linear function of the lateral loadings, that is,

$$\begin{aligned} y_1 &= h_{11}Y_1 + h_{12}Y_2 + h_{13}Y_3 \cdots, \\ y_2 &= h_{21}Y_1 + h_{22}Y_2 + h_{23}Y_3 \cdots, \\ z_1 &= h_{11}Z_1 + h_{12}Z_2 + h_{13}Z_3 \cdots, \\ z_2 &= h_{21}Z_1 + h_{22}Z_2 + h_{23}Z_3 \cdots, \end{aligned}$$

where

$$\begin{aligned} Y_1 = -\,m_1(\ddot{y}_1 - h_1(\omega^2 + 2\omega\dot{\epsilon}_1)\cos(\omega t + \phi_{01}) \\ - \ddot{\epsilon}_1h_1 \sin(\omega t + \phi_{01})), \end{aligned}$$

$$\begin{aligned} Z_1 = -\,m_1(\ddot{z}_1 - h_1(\omega^2 + 2\omega\dot{\epsilon}_1)\sin(\omega t + \phi_{01}) \\ - \ddot{\epsilon}_1h_1 \cos(\omega t + \phi_{01}), \text{ etc.} \end{aligned}$$

The moment equations retain their previous form.

When h is small, in a first approximation and in view of neglecting gyroscopic reactions, we are quite justified in also neglecting the torsional effect on the lateral vibrations. The equations simplify then to

$$\begin{aligned} y_1 = & - m_1h_{11}(\ddot{y}_1 - h_1\omega^2 \cos (\omega t + \phi_{01})) \\ & \qquad - m_2h_{12}(\ddot{y}_2 - h_2\omega^2 \cos (\omega t + \phi_{02}) \\ & \qquad\qquad - m_3h_{13}(\ddot{y}_3 - h_3\omega^2 \cos (\omega t + \phi_{03})) \cdots, \\ y_2 = & - m_1h_{21}(\ddot{y}_1 - h_1\omega^2 \cos (\omega t + \phi_{01})) \\ & \qquad - m_2h_{22}(\ddot{y}_2 - h_2\omega^2 \cos (\omega t + \phi_{02})) \\ & \qquad\qquad - m_3h_{23}(\ddot{y}_3 - h_3\omega^2 \cos (\omega t + \phi_{03})), \\ z_1 = & - m_1h_{11}(\ddot{z}_1 - h_1\omega^2 \sin (\omega t + \phi_{01})) \\ & \qquad - m_2h_{12}(\ddot{z}_2 - h_2\omega^2 \sin (\omega t + \phi_{02})) \\ & \qquad\qquad - m_3h_{13}(\ddot{z}_3 - h_3\omega^2 \sin (\omega t + \phi_{03})), \\ z_2 = & - m_1h_{21}(\ddot{z}_1 - h_1\omega^2 \sin (\omega t + \phi_{01}) \\ & \qquad - m_2h_{22}(\ddot{z}_2 - h_2\omega^2 \sin (\omega t + \phi_{02})) \\ & \qquad\qquad - m_3h_{23}(\ddot{z}_3 - h_3\omega^2 \sin (\omega t + \phi_{03})). \end{aligned}$$

These equations may be written in the form

$$m_1h_1\ddot{y}_1 + m_2h_{12}\ddot{y}_2 + m_3h_{13}\ddot{y}_3 - - + y_1 = \omega^2 Q \cos (\omega t + \gamma_1),$$

$$m_2h_{21}\ddot{y}_1 + m_2h_{22}\ddot{y}_2 + m_3h_{23}\ddot{y}_3 - - + y_2 = \omega^2 R \cos (\omega t + \gamma_2),$$

. . . .

with similar equations for z_1. Evidently

$$\begin{aligned} Q \cos (\omega t + \gamma) = & \; m_1h_{11}h_1 \cos (\omega t + \phi_{01}) \\ & \qquad + m_2h_{12}h_2 \cos (\omega t + \phi_{02}), \end{aligned}$$

$$\tan \gamma_1 = \frac{m_1h_{11}h_1 \sin \phi_{01} + m_2h_{12}h_2 \sin \phi_{02}}{m_1h_{11}h_1 \cos \phi_{01} + m_2h_{12}h_2 \cos \phi_{02}}, \text{ etc.}$$

The condition for the whirling speeds is when the determinate,

$$\begin{vmatrix} m_1h_{11}\omega^2 - 1 & m_2h_{12}\omega^2 & m_3h_{13}\omega^2 \\ m_2h_{21}\omega^2 & m_2h_{22}\omega^2 - 1 & m_3h_{23}\omega^2 \\ m_3h_{31}\omega^2 & m_3h_{32}\omega^2 & m_3h_{33}\omega^2 - 1 \end{vmatrix} = 0.$$

It is to be noted that with elastic bearings of different equivalent spring constants in the horizontal and vertical planes, the a's and h's are no longer the same as coefficients of y and z. Thus additional critical speeds will be obtained due to the necessity of considering both the y and z equations.

RELATIONS OF THE ELASTIC COEFFICIENTS.

The potential energy of the shaft due to the lateral loadings in terms of the displacements has the quadratic form,

$$V = \tfrac{1}{2}[a_{11}y_1^2 + a_{22}y_2^2 + a_{33}y_3^2 + 2a_{12}y_1y_2 \\ + 2a_{23}y_2y_3 + 2a_{31}y_3y_1 \cdots],$$

where we note the reciprocal relations $a_{12} = a_{21}$, $a_{23} = a_{32}$, etc., the loadings have the linear form,

$$P_1 = a_{11}y_1 + a_{12}y_2 + a_{23}y_3 \cdots = \frac{\partial V}{\partial y_1},$$

$$P_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 \cdots = \frac{\partial V}{\partial y_2},$$

$$P_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3 \cdots = \frac{\partial V}{\partial y_3}.$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

The potential energy, however, may also be expressed in terms of the loadings and the redundant reactions, which can be considered as "reaction" coördinates.

On Castigliano's theorem:

If we express the elastic potential in terms of the loadings and the redundant reactions acting on the system, then for small displacements, if $P_1 \cdots P_n$ are the applied forces and $R_1 \cdots R_n$ are the redundant reactions and moments,

$$V = f(P_1 \cdots P_n, R_1 \cdots R_m),$$

so that

$$\delta V = \frac{\partial V}{\partial P_1}\delta P_1 \cdots + \frac{\partial V}{\partial P_n}\delta P_n + \frac{\partial V}{\delta R_1}\partial R_1 \cdots + \frac{\partial V}{\partial R_m}\delta R_m, \quad (1)$$

where otherwise,

$$V = \tfrac{1}{2}(P_1y_1 \cdots P_ny_n + R_1n_1 \cdots R_mn_m)$$

and

$$\delta V = \tfrac{1}{2}[P_1\delta y_1 + y_1\delta P_1 \cdots + P_n\delta y_n + y_n\delta P_n \\ + R_1\delta n_1 + n_1\delta R_1 \cdots + R_m\delta n_m + n_m\delta R_m]. \quad (2)$$

But also

$$\delta V = P_1\delta y_1 \cdots + P_n\delta y_n + R_1\delta n_1 \cdots + R_m\delta n_m. \tag{3}$$

Therefore, combining (2) and (3),

$$\delta V = y_1\delta P_1 \cdots + y_n\delta P_n + n_1\delta R_1 \cdots + n_m\delta R_m. \tag{4}$$

On equating (1) and (4) we obtain

$$\left(\frac{\partial V}{\partial P_1} - y_1\right)\delta P_1 \cdots + \left(\frac{\partial V}{\partial P_n} - y_n\right)\delta P_n + \left(\frac{\partial V}{\partial R_1} - n_1\right)\delta R_1 \cdots + \left(\frac{\partial V}{\partial R_m} - n_m\right)\delta R_m = 0. \tag{5}$$

Since the loadings and redundant reactions are independent, then for any arbitrary variations of the loadings and redundant reactions their coefficients must be nil, so that,

$$y_1 = \frac{\partial V}{\partial P_1}, \qquad y_n = \frac{\partial V}{\partial P_n}, \qquad n_1 = \frac{\partial V}{\partial R_n}, \qquad n_m = \frac{\partial V}{\partial R_m}.$$

For constraint redundant reactions,

$$\delta V = y_1\delta P_1 \cdots + y_n\delta P_n$$

and

$$\delta V = \frac{\partial V}{\partial P_1}\delta P_1 \cdots + \frac{\partial V}{\partial P_n}\delta P_n + \frac{\partial V}{\partial R_1}\delta R_1 \cdots + \frac{\partial V}{\partial R_m}\delta R_m,$$

so that

$$\left(\frac{\partial V}{\partial P_1} - y_1\right)\delta P_1 \cdots + \left(\frac{\partial V}{\partial P_n} - y_n\right)\delta P_n + \frac{\partial V}{\partial R_1}\delta R_1 \cdots + \frac{\partial V}{\partial R_m}\delta R_m = 0,$$

and

$$y_1 = \frac{\partial V}{\partial P_1}, \qquad y_n = \frac{\partial V}{\partial P_n}, \qquad \frac{\partial V}{\partial R_1} = 0, \qquad \frac{\partial V}{\partial R_m} = 0.$$

Slope Deflection Theorem

A very simple application is the slope deflection equation of a beam,—

We have the statical relation,

$$M_A - M_B + Sl = 0,$$

where M_A and M_B are the constraining moments at the ends and S is the shear reaction, θ_A and θ_B are the angular deflections at A and B and y the relative deflection of B with respect to A.

From the statical relation above, the reactions M_A, M_B, and S are not independent, so that V can be expressed as a function of any two of the reaction coördinates.

$$\therefore \quad \delta V = \frac{\partial V}{\partial S}\delta S + \frac{\partial V}{\partial M_A}\delta M_A,$$

but

$$\begin{aligned} \delta V &= -\delta S\cdot y - \delta M_A\cdot\theta_A + \delta M_B\cdot\theta_B \\ &= -\delta S\cdot y - \delta M_A\cdot\theta_A + \left(\frac{\partial M_B}{\partial M_A}\delta M_A + \frac{\partial M_B}{\partial S}\cdot\delta S\right)\theta_B \\ &= -\delta S\cdot y + \left(\frac{\partial M_B}{\partial M_A}\cdot\theta_B - \theta_A\right)\delta M_A + \frac{\partial M_B}{\partial S}\cdot\theta_B\cdot\delta S \\ &= \left(\frac{\partial M_B}{\partial S}\theta_B - y\right)\delta S + \left(\frac{\partial M_B}{\partial M_A}\theta_B - \theta_A\right)\delta M_A. \end{aligned}$$

Therefore,

$$\frac{\partial V}{\partial S} = \frac{\partial M_B}{\partial S}\cdot\theta_B - y, \qquad \text{and} \qquad \frac{\partial V}{\partial M_A} = \frac{\partial M_B}{\partial M_A}\cdot\theta_B - \theta_A,$$

but since

$$M_B = M_A + Sl, \qquad \frac{\partial M_B}{\partial M_A} = 1, \qquad \frac{\partial M_B}{\partial S} = l;$$

$$\therefore \quad \frac{\partial V}{\partial S} = l\theta_B - y, \qquad \frac{\partial V}{\partial M_A} = \theta_B - \theta_A.$$

The bending moment, for any section is $M_A = M + Sx$ so that

$$\frac{\partial M}{\partial M_A} = 1, \qquad \frac{\partial M}{\partial S} = x.$$

We have, then,

$$\frac{\partial V}{\partial S} = \int \frac{M}{EI}\frac{\partial M}{\partial S}\cdot dx = \int \frac{Mxdx}{EI} = l\theta_B - y;$$

$$\therefore \quad y = l\theta_B - \int \frac{Mxdx}{EI},$$

$$\theta_B - \theta_A = \int \frac{Mdx}{EI}.$$

The above shows the process of analysis required in the more complex problems in the application of Castigliano's theorem.

In the procedure of a shafting problem, with applied loads $P_1P_2P_3 \cdots$ and redundant bearing reactions, $R_mR_n \cdots$ the potential energy has the quadratic form,

$$\begin{aligned} V = \tfrac{1}{2}(&b_{11}P_1^2 + b_{22}P_2^2 + b_{33}P_3^2 \cdots + C_{mm}R_m^2 + C_{nn}R_n^2 \\ &+ 2b_{12}P_1P_2 + 2b_{23}P_2P_3 + 2b_{31}P_3P_1 \cdots \\ &+ 2C_{1m}P_1R_m + 2C_{2m}P_2R_m + 2C_{3m}P_3R_m \cdots \\ &+ 2C_{1n}P_1R_n + 2C_{2n}P_2R_n + 2C_{3n}P_3R_n \cdots C_{mn}R_mR_n \cdots). \end{aligned}$$

The redundant reactions are obtained from the conditions,

$$\frac{\partial V}{\partial R_m} = C_{mm}R_m + C_{mn}R_n + C_{1m}P_1 + C_{2m}P_2 + C_{3m}P_3 = 0,$$

$$\frac{\partial V}{\partial R_n} = C_{nn}R_n + C_{mn}R_m + C_{1n}P_1 + C_{2n}P_2 + C_{3n}P_3 = 0,$$

or otherwise, in an actual calculation,

$$\frac{\partial V}{\partial R_m} = \Sigma \int \frac{M}{EI}\frac{\partial M}{\partial R_m} = 0, \qquad \frac{\partial V}{\partial R_n} = \Sigma \int \frac{M}{EI}\frac{\partial M}{\partial R_n} = 0.$$

Where the bending moment M is expressed in terms of the loadings and redundant reactions.

From these equations we obtain the redundant reactions as linear functions of P_1P_2 etc. Thus for the case of two redundant reactions,

$$R_m = k_{1m}P_1 + k_{2m}P_2 + k_{3m}P_3,$$

$$R_n = j_{1n}P_1 + j_{2n}P_2 + j_{3n}P_3,$$

where

$$k_{1m} = \frac{C_{1m}C_{nn} - C_{1n}C_{mn}}{C_{mn}^2 - C_{mm}C_{nn}}, \qquad k_{2m} = \frac{C_{2m}C_{nn} - C_{2n}C_{mn}}{C_{mn}^2 - C_{mm}C_{nn}}, \text{ etc.}$$

$$j_{1n} = \frac{C_{1n}C_{mm} - C_{1m}C_{mn}}{C_{mn}^2 - C_{mm}C_{nn}}, \qquad j_{2n} = \frac{C_{2n}C_{mm} - C_{2m}C_{mn}}{C_{mn}^2 - C_{mm}C_{nn}}, \text{ etc.}$$

The deflections at the loadings $P_1P_2P_3 \cdots$ are,

$$y_1 = \frac{\partial V}{\partial P_1}, \qquad y_2 = \frac{\partial V}{\partial P_2}, \qquad y_3 = \frac{\partial V}{\partial P_3},$$

or

$$\begin{aligned} y_1 &= b_{11}P_1 + b_{12}P_2 + b_{13}P_3 + C_{1m}R_m + C_{1n}R_n, \\ y_2 &= b_{21}P_1 + b_{22}P_2 + b_{23}P_3 + C_{2m}R_m + C_{2n}R_n, \\ y_3 &= b_{31}P_1 + b_{32}P_2 + b_{33}P_3 + C_{3m}R_m + C_{3n}R_n. \end{aligned}$$

Eliminating the redundant reactions, we have

$$\begin{aligned} y_1 &= h_{11}P_1 + h_{12}P_2 + h_{13}P_3, \\ y_2 &= h_{21}P_1 + h_{22}P_2 + h_{23}P_3, \\ y_3 &= h_{31}P_1 + h_{32}P_2 + h_{33}P_3, \end{aligned}$$

where

$$h_{11} = b_{11} + C_{1m}k_{1m} + C_{1n}j_{1n},$$

$$h_{12} = b_{12} + C_{1m}k_{2m} + C_{1n}j_{2n} = b_{21} + C_{2m}k_{1m} + C_{2n}j_{1m} = h_{21};$$

$\therefore \quad h_{12} = h_{21}$, in like manner, $h_{23} = h_{32}$, etc.

The relations $h_{12} = h_{21}$, $h_{23} = h_{32}$ etc. are also evident from a direct application of the reciprocal theorem.

In the case of a continuous system, for redundant reactions between any two parts, we note from the principle of action and reaction between the parts and the common displacement of the interface dividing the parts that,

$$\frac{\partial V_1}{\partial R} = \delta, \qquad \frac{\partial V_2}{\partial R} = -\delta,$$

where $V = V_1 + V_2$.

$$\therefore \quad \frac{\partial V_1}{\partial R} + \frac{\partial V_2}{\partial R} = \frac{\partial V}{\partial R} = 0$$

for the redundant reaction R between the parts.

In the consideration of elastic bearing supports, we have additional terms for the potential in the form,

$$\tfrac{1}{2}C_m y_m{}^2 = \tfrac{1}{2}\frac{R^2{}_m}{C_m}, \qquad \tfrac{1}{2}C_n y_n{}^2 = \tfrac{1}{2}\frac{R^2{}_n}{C_n}, \text{ etc.}$$

Treating the shaft and springs of the supports as a continuous system, the redundant reactions are obtained from,

$$\frac{\partial V}{\partial R_m} = \left(\frac{1}{C_m} + C_{mm}\right)R_m + C_{mn}R_n$$
$$+ C_{1m}P_1 + C_{2m}P_2 + C_{3m}P_3 = 0,$$

$$\frac{\partial V}{\partial R_n} = \left(\frac{1}{C_n} + C_{nn}\right)R_n + C_{nm}R_m$$
$$+ C_{1n}P_1 + C_{2n}P_2 + C_{3n}P_3 = 0.$$

The displacement at the loadings are

$$y_1 = \frac{\partial V}{\partial P_1}, \qquad y_2 = \frac{\partial V}{\partial P_2}, \qquad y_3 = \frac{\partial V}{\partial P_3}.$$

Substituting for the redundant reactions, we again have the displacements as a linear function of the loadings, i.e.,

$$\begin{aligned} y_1 &= h_{11}P_1 + h_{12}P_2 + h_{13}P_3, \\ y_2 &= h_{21}P_1 + h_{22}P_2 + h_{23}P_3, \\ y_3 &= h_{31}P_1 + h_{32}P_2 + h_{33}P_3. \end{aligned}$$

By a simple transformation we may finally express the loadings as a linear function of the displacements, i.e.,

$$\begin{aligned} P_1 &= a_{11}y_1 + a_{12}y_2 + a_{13}y_3, \\ P_2 &= a_{21}y_1 + a_{22}y_2 + a_{23}y_3, \\ P_3 &= a_{31}y_1 + a_{32}y_2 + a_{33}y_3, \end{aligned}$$

where

$$a_{11} = \frac{\begin{vmatrix} h_{22}h_{23} \\ h_{32}h_{33} \end{vmatrix}}{D}, \qquad a_{22} = \frac{\begin{vmatrix} h_{11}h_{13} \\ h_{31}h_{33} \end{vmatrix}}{D}, \qquad a_{33} = \frac{\begin{vmatrix} h_{11}h_{12} \\ h_{21}h_{22} \end{vmatrix}}{D},$$

$$a_{12} = \frac{\begin{vmatrix} h_{12}h_{13} \\ h_{32}h_{33} \end{vmatrix}}{D}, \qquad a_{23} = \frac{\begin{vmatrix} h_{11}h_{12} \\ h_{31}h_{32} \end{vmatrix}}{D}, \qquad a_{31} = \frac{\begin{vmatrix} h_{21}h_{22} \\ h_{31}h_{32} \end{vmatrix}}{D},$$

where

$$D = \begin{vmatrix} h_{11}h_{12}h_{13} \\ h_{21}h_{22}h_{23} \\ h_{31}h_{32}h_{33} \end{vmatrix}.$$

This procedure can be extended to further degrees of freedom.

ENERGY METHOD IN SHAFT ANALYSIS.

It is well known that the whirling speed of a shaft corresponds to the natural frequency of vibration for lateral vibrations. Good approximations for the fundamental frequency are obtained by assuming an arbitrary deflection curve of the shaft and equating the maximum strain energy in its deflected position to the maximum kinetic energy in its neutral position. This method was introduced by Rayleigh, and Morely applied it to the shaft problem, assuming the deflected curve to correspond to its static deflected position.

When, however, this method is applied to disks subjected to angular vibrations sufficiently small so as not to affect the deflected curve corresponding to lateral vibrations alone, the natural frequency is found to be lowered whereas actually the whirling speed is raised.

If the rotational speed is included in the expression of the kinetic energy for a deflected disk, Chree regarded the whirling speed as that speed for which the frequency of vibration becomes nil, that is, the critical speed at which the shaft ceases to vibrate. Lorenz, Föppl, Lees, and others consider the critical speed with a slightly unbalanced disk simply as a condition of resonance, that is, when the forced vibration resulting from the centrifugal loading coincides with the natural frequency.

We will regard the whirling speed as a condition of kinetic instability. The condition of stability approximating the critical speed means that the total input into the shaft and the work of the internal elastic reactions do not exceed the rotational kinetic energy at the maximum deflected position. If this work exceeds the rotational kinetic energy at the deflected configuration we have the condition of instability. Therefore, the whirling or critical speed corresponds to that speed at which the work done on the shaft from its neutral or undeflected position to its deflected position, including the corresponding change in the elastic potential, is just equal to the rotational kinetic energy alone in the deflected position. We may regard the work from the neutral to the deflected position to occur approximately at the critical speed.

At first, disregarding the angular turning of the planes of the disks, we have

$$\sum \frac{d(m\omega y^2)}{dt}\cdot\omega\cdot dt = dV + \sum d\left(\frac{1}{2}m\omega^2 y^2 + \frac{1}{2}m\dot{y}^2\right),$$

where the first term is the work of the impressed torque and V is the potential energy of the shaft. Then,

$$\sum \omega\cdot d(m\omega y^2) = dV + \sum m(\omega^2 y\,dy + y^2\omega\cdot d\omega + \dot{y}\,d\dot{y}),$$

$$\sum m(2\omega^2 y\,dy + y^2\omega\,d\omega) = dV + \sum m(\omega^2 y\,dy + y^2\omega\,d\omega + \dot{y}\,d\dot{y});$$

$$\therefore \quad \sum m\omega^2 y\,dy = dV + \sum m\dot{y}\,d\dot{y},$$

and the total work from the neutral to the deflected position, noting the lateral velocity is zero at the extreme positions, is

$$\int_0^y \sum m\omega^2 y\,dy = \int_0^V dV + \int_0^0 \sum m\dot{y}\,d\dot{y};$$

$$\therefore \frac{\sum m\omega^2 y^2}{2} = V = \frac{\sum mgy}{2} \text{ (approx.)}.$$

That is, the work of the centrifugal forces equals the elastic potential energy at its deflected position.

If now we consider the angular deflections of the plane of the disks and assume the deflection to occur at synchronous precession, and if k_0 = the polar radius of gyration of any disk and k its radius of gyration about a diameter, then for a deflection θ for any disk, the angular momentum about the shaft axis is approximately,

$$\begin{aligned}\text{A.M.} &= \sum mk_0^2\omega\cos^2\theta + \sum mk^2\omega\sin^2\theta + \sum m\omega y^2\\ &= \sum m(k_0^2 - (k_0^2 - k^2)\sin^2\theta)\omega + \sum m\omega y^2.\end{aligned}$$

The impressed torque on the shaft is, therefore,

$$\Phi = -2\sum m\omega(k_0^2 - k^2)\sin\theta\cdot\cos\theta\cdot\dot{\theta} + \sum\frac{d(m\omega y^2)}{dt}.$$

The change in kinetic energy from its neutral to its deflected position may be estimated as follows. The kinetic energy at the neutral position is simply $\frac{1}{2} \sum mk_0{}^2\omega^2$. At the deflected position the kinetic energy is,

$$\tfrac{1}{2} \sum m\omega^2y^2 + \tfrac{1}{2} \sum mk_0{}^2\omega^2 \cos^2 \theta + \tfrac{1}{2} \sum mk^2\omega^2 \sin^2 \theta.$$

The change of kinetic energy is

$$\tfrac{1}{2} \sum m\omega^2(y^2 - (k_0{}^2 - k^2) \sin^2 \theta).$$

The equation of energy between the neutral and deflected positions is, therefore,

$$\int \Phi\omega dt - V = \frac{1}{2} \sum m\omega^2(y^2 - (k_0{}^2 - k^2) \sin^2 \theta).$$

But since,

$$\begin{aligned} \int \Phi\omega dt &= -2\int_0^\theta \sum m\omega^2(k_0{}^2 - k^2) \sin \theta \cos \theta d\theta + \int_0^y \sum d(m\omega^2y^2)\cdot\omega \\ &= -\sum m\omega^2(k_0{}^2 - k^2) \sin^2 \theta + \sum m\omega^2y^2; \\ \therefore \quad & \tfrac{1}{2}\omega^2 \sum m(y^2 - (k_0{}^2 - k^2) \sin^2 \theta) = V. \end{aligned}$$

For small displacements $\sin^2 \theta \doteq \theta^2$, and for circular disks $k_0{}^2 = 2k^2$, and if V is measured by the potential energy corresponding to the static deflection curve,

$$\tfrac{1}{2}\omega^2 \sum m(y^2 - k^2\theta^2) = \tfrac{1}{2} \sum mgy.$$

Hence the whirling speed is given by

$$\omega^2 = \frac{g \sum my}{\sum m(y^2 - k^2\theta^2)}.$$

On the other hand the natural frequency is

$$p^2 = \frac{g \sum my}{\sum m(y^2 + k^2\theta^2)},$$

and neglecting the gyroscopic effect,

$$p^2 = \omega^2 = \frac{g \sum my}{\sum my^2}.$$

ON INERTIA COEFFICIENTS AND VELOCITY RATIOS.

The kinetic energy of a dynamical system is a homogeneous quadratic function of the independent coördinates. Thus, for two degrees of freedom,

$$T = \tfrac{1}{2}(a_{11}\phi_1{}^2 + a_{22}\phi_2{}^2 + 2a_{12}\phi_1\phi_2),$$

where the coefficients of inertias expressed in partial velocity ratios have the form,

$$\begin{aligned} a_{11} &= \sum M(K^2{}_{x\phi_1} + K^2{}_{y\phi_1}) + \sum IK^2{}_{\psi\phi_1}, \\ a_{22} &= \sum M(K^2{}_{x\phi_2} + K^2{}_{y\phi_2}) + \sum IK^2{}_{\psi\phi_2}, \\ a_{12} &= a_{21} = \sum M(K_{x\phi_1}K_{x\phi_2} + K_{y\phi_1}K_{y\phi_2}) + \sum IK_{\psi\phi_1}K_{\psi\phi_2} \end{aligned}$$

where

$$K_{x\phi_1} = \frac{\partial x}{\partial \phi_1}, \quad K_{y\phi_1} = \frac{\partial y}{\partial \phi_1} \quad \text{and} \quad K_{y\phi_2} = \frac{\partial x}{\partial \phi_2}, \quad K_{y\phi_2} = \frac{\partial y}{\partial \phi_2}.$$

In a dynamical system, we have a variety of inertia distributions. The *M*'s may refer to the total mass of a member and *I*, its moment of inertia about its c. of g., so that the velocity ratios are derived from the coördinates of the c. of g.'s and the angular displacements of the members. Or we may regard the *M*'s as the weighted loads at the joints or pins and the moment of inertia $I = M(k_g{}^2 - z(l - z))$, where z is the distance of the c. of g. from either pin and k_g is the radius of gyration about the c. of g. For this case the velocity ratios are referred to the pin joints and the angular motion of the members.

Let us assume generalized forces applied to the system of the nature,

$$\begin{aligned} \Phi_1 &= \sum(XK_{x\phi_1} + YK_{y\phi_1}) + \sum LK_{\psi\phi_1} - \frac{\partial V}{\partial \phi_1}, \\ \Phi_2 &= \sum(XK_{x\phi_2} + YK_{y\phi_2}) + \sum LK_{\psi\phi_2} - \frac{\partial V}{\partial \phi_2}, \end{aligned}$$

where the *L*'s are the applied torques or moments and *V* is the potential function not otherwise included in the applied loading. It is to be noted reactions of constraints and internal reactions which have common displacements as in rigid bodies, are excluded.

From the Lagrangian equations, noting the coefficients of inertias themselves are functions of the independent coördinates, we have for two degrees of freedom, the equations of motion expressed in terms of the inertia coefficients,

$$\begin{aligned} a_{11}\ddot{\phi}_1 + a_{12}\ddot{\phi}_2 &+ \frac{1}{2}\frac{\partial a_{11}}{\partial \phi_1}\dot{\phi}_1^2 \\ &+ \left(\frac{\partial a_{12}}{\partial \phi_2} - \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_1}\right)\dot{\phi}_2^2 + \frac{\partial a_{11}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2 = \Phi_1, \end{aligned}$$

$$\begin{aligned} a_{22}\ddot{\phi}_2 + a_{21}\ddot{\phi}_1 &+ \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_2}\dot{\phi}_2^2 \\ &+ \left(\frac{\partial a_{12}}{\partial \phi_1} - \frac{1}{2}\frac{\partial a_{11}}{\partial \phi_2}\right)\dot{\phi}_1^2 + \frac{\partial a_{22}}{\partial \phi_1}\dot{\phi}_1\dot{\phi}_2 = \Phi_2. \end{aligned}$$

The form of these equations are the basis of the Christoffel notation. We wish to show, however, that the equations in terms of partial velocity ratios previously derived from D'Alembert's principle, can also be derived from a consideration of the inertia coefficients in these equations. Since

$$\begin{aligned} a_{11} &= \sum M(K^2{}_{x\phi_1} + K^2{}_{y\phi_1}) + \sum I K^2{}_{\psi\phi_1}, \\ a_{22} &= \sum M(K^2{}_{x\phi_2} + K^2{}_{y\phi_2}) + \sum I K^2{}_{\psi\phi_2}, \\ a_{12} &= a_{21} = \sum M(K_{x\phi_1}K_{x\phi_2} + K_{y\phi_1}K_{y\phi_2}) + \sum I K_{\psi\phi_1}K_{\psi\phi_2} \end{aligned}$$

we have

$$\begin{aligned} \frac{\partial a_{12}}{\partial \phi_2} &= \frac{\partial}{\partial \phi_2}\sum M(K_{x\phi_1}K_{x\phi_2} + K_{y\phi_1}K_{y\phi_2}) + \frac{\partial}{\partial \phi_2}\sum I K_{\psi\phi_1}K_{\psi\phi_2} \\ &= \sum M\left[\left(K_{x\phi_1}\frac{\partial K_{x\phi_2}}{\partial \phi_2} + K_{x\phi_2}\frac{\partial K_{x\phi_1}}{\partial \phi_2}\right)\right. \\ &\quad \left. + \left(K_{y\phi_1}\frac{\partial K_{y\phi_2}}{\partial \phi_2} + K_{y\phi_2}\frac{\partial K_{y\phi_1}}{\partial \phi_2}\right)\right] \\ &\quad + \sum I\left[\left(K_{\psi\phi_1}\frac{\partial K_{\psi\phi_2}}{\partial \phi_2} + K_{\psi\phi_2}\frac{\partial K_{\psi\phi_1}}{\partial \phi_2}\right)\right], \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_1} &= \frac{1}{2}\frac{\partial}{\partial \phi_1}\sum M(K^2{}_{x\phi_2} + K^2{}_{y\phi_2}) + \frac{1}{2}\frac{\partial}{\partial \phi_1}\sum I K^2{}_{\psi\phi_2} \\ &= \sum M\left(K_{x\phi_2}\frac{\partial K_{x\phi_2}}{\partial \phi_1} + K_{y\phi_2}\frac{\partial K_{y\phi_2}}{\partial \phi_1}\right) + \sum I K_{\psi\phi_2}\frac{\partial K_{\psi\phi_2}}{\partial \phi_1}. \end{aligned}$$

Since, however,

$$\frac{\partial K_{x\phi_1}}{\partial \phi_2} = \frac{\partial K_{x\phi_2}}{\partial \phi_1} = \frac{\partial^2 x}{\partial \phi_1 \partial \phi_2}, \qquad \frac{\partial K_{y\phi_1}}{\partial \phi_2} = \frac{\partial K_{y\phi_2}}{\partial \phi_1} = \frac{\partial^2 y}{\partial \phi_1 \partial \phi_2}$$

and

$$\frac{\partial K_{\psi\phi_1}}{\partial \phi_2} = \frac{\partial K_{\psi\phi_2}}{\partial \phi_1} = \frac{\partial^2 \psi}{\partial \phi_1 \partial \phi_2}$$

we have

$$\frac{\partial a_{12}}{\partial \phi_2} - \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_1} = \sum M\left(K_{x\phi_1}\frac{\partial K_{x\phi_2}}{\partial \phi_2} + K_{y\phi_1}\frac{\partial K_{y\phi_2}}{\partial \phi_2}\right) + \sum I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_2}}{\partial \phi_2}.$$

In like manner,

$$\frac{\partial a_{12}}{\partial \phi_1} - \frac{1}{2}\frac{\partial a_{11}}{\partial \phi_2} = \sum M\left(K_{x\phi_2}\frac{\partial K_{x\phi_1}}{\partial \phi_1} + K_{y\phi_2}\frac{\partial K_{y\phi_1}}{\partial \phi_1}\right) + \sum I K_{\psi\phi_2}\frac{\partial K_{\psi\phi_1}}{\partial \phi_1}.$$

Also,

$$\frac{1}{2}\frac{\partial a_{11}}{\partial \phi_1} = \sum M\left(K_{x\phi_1}\frac{\partial K_{x\phi_1}}{\partial \phi_1} + K_{y\phi_1}\frac{\partial K_{y\phi_1}}{\partial \phi_1}\right) + \sum I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_1}}{\partial \phi_1},$$

$$\frac{\partial a_{11}}{\partial \phi_2} = 2\sum M\left(K_{x\phi_1}\frac{\partial K_{x\phi_1}}{\partial \phi_2} + K_{y\phi_1}\frac{\partial K_{y\phi_1}}{\partial \phi_2}\right) + 2\sum I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_1}}{\partial \phi_2}.$$

and

$$\frac{1}{2}\frac{\partial a_{22}}{\partial \phi_2} = \sum M\left(K_{x\phi_2}\frac{\partial K_{x\phi_2}}{\partial \phi_2} + K_{y\phi_2}\frac{\partial K_{y\phi_2}}{\partial \phi_2}\right) + \sum I K_{\psi\phi_2}\frac{\partial K_{\psi\phi_2}}{\partial \phi_2},$$

$$\frac{\partial a_{22}}{\partial \phi_1} = 2\sum M\left(K_{x\phi_2}\frac{\partial K_{x\phi_2}}{\partial \phi_1} + K_{y\phi_2}\frac{\partial K_{y\phi_2}}{\partial \phi_1}\right) + 2\sum I K_{\psi\phi_2}\frac{\partial K_{\psi\phi_2}}{\partial \phi_1}.$$

On substituting in the equations of motion in terms of the inertia coefficients, we obtain,

$$\sum M K_{x\phi_1}\Big[K_{x\phi_1}\ddot{\phi}_1 + K_{x\phi_2}\ddot{\phi}_2 + \frac{\partial K_{x\phi_1}}{\partial \phi_1}\phi_1{}^2 + \frac{\partial K_{x\phi_2}}{\partial \phi_2}\phi_2{}^2$$

$$+ 2\frac{\partial K_{x\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2\Big] + \sum MK_{y\phi_1}\Big[K_{y\phi_1}\ddot{\phi}_1 + K_{y\phi_2}\ddot{\phi}_2$$

$$+ \frac{\partial K_{y\phi_1}}{\partial \phi_1}\dot{\phi}_1^2 + \frac{\partial K_{y\phi_2}}{\partial \phi_2}\dot{\phi}_2^2 + 2\frac{\partial K_{y\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2\Big]$$

$$+ \sum IK_{\psi\phi_1}\Big[K_{\psi\phi_1}\ddot{\phi}_1 + K_{\psi\phi_2}\ddot{\phi}_2 + \frac{\partial K_{\psi\phi_1}}{\partial \phi_1}\dot{\phi}_1^2$$

$$+ \frac{\partial K_{\psi\phi_2}}{\partial \phi_2}\dot{\phi}_2^2 + 2\frac{\partial K_{\psi\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2\Big] = \Phi_1$$

and

$$\sum MK_{x\phi_2}\Big[K_{x\phi_1}\ddot{\phi}_1 + K_{x\phi_2}\ddot{\phi}_2 + \frac{\partial K_{x\phi_1}}{\partial \phi_1}\dot{\phi}_1^2 + \frac{\partial K_{x\phi_2}}{\partial \phi_2}\dot{\phi}_2^2$$

$$+ 2\frac{\partial K_{x\phi_2}}{\partial \phi_1}\dot{\phi}_1\dot{\phi}_2\Big] + \sum MK_{y\phi_2}\Big[K_{y\phi_1}\ddot{\phi}_1 + K_{y\phi_2}\ddot{\phi}_2$$

$$+ \frac{\partial K_{y\phi_1}}{\partial \phi_1}\dot{\phi}_1^2 + \frac{\partial K_{y\phi_2}}{\partial \phi_2}\dot{\phi}_2^2 + 2\frac{\partial K_{y\phi_2}}{\partial \phi_1}\dot{\phi}_1\dot{\phi}_2\Big]$$

$$+ \sum IK_{\psi\phi_2}\Big[K_{\psi\phi_1}\ddot{\phi}_1 + K_{\psi\phi_2}\ddot{\phi}_2 + \frac{\partial K_{\psi\phi_1}}{\partial \phi_1}\dot{\phi}_1^2$$

$$+ \frac{\partial K_{\psi\phi_2}}{\partial \phi_2}\dot{\phi}_2^2 + 2\frac{\partial K_{\psi\phi_2}}{\partial \phi_1}\dot{\phi}_1\dot{\phi}_2\Big] = \Phi_2,$$

where

$$\frac{\partial K_{x\phi_1}}{\partial \phi_2} = \frac{\partial K_{x\phi_2}}{\partial \phi_1} = \frac{\partial^2 x}{\partial \phi_1 \partial \phi_2}, \text{ etc.}$$

In a similar way for three degrees of freedom,

$$T = \tfrac{1}{2}(a_{11}\dot{\phi}_1^2 + a_{22}\dot{\phi}_2^2 + a_{33}\dot{\phi}_3^2 + 2a_{12}\dot{\phi}_1\dot{\phi}_2 + 2a_{23}\dot{\phi}_2\dot{\phi}_3 + 2a_{31}\dot{\phi}_3\dot{\phi}_1)$$

for the kinetic energy in terms of the inertia coefficients, where as before,

$$a_{23} = a_{32} = \sum M(K_{x\phi_2}K_{x\phi_3} + K_{y\phi_2}K_{y\phi_3}) + \sum I(K_{\psi\phi_2}K_{\psi\phi_3} + K_{\psi\phi_2}K_{\psi\phi_3})$$

$$a_{33} = \sum M(K^2_{x\phi_3} + K^2_{y\phi_3}) + \sum IK^2_{\psi\phi_3}, \text{ etc.}$$

From the Lagrangian equations for the ϕ_1 coördinate, noting the inertia coefficients are functions of the coördinates,

$$a_{11}\ddot{\phi}_1 + a_{12}\ddot{\phi}_2 + a_{13}\ddot{\phi}_3 + \frac{1}{2}\frac{\partial a_{11}}{\partial \phi_1}\dot{\phi}_1^2 + \left(\frac{\partial a_{12}}{\partial \phi_2} - \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_1}\right)\dot{\phi}_2^2$$
$$+ \left(\frac{\partial a_{13}}{\partial \phi_3} - \frac{1}{2}\frac{\partial a_{33}}{\partial \phi_1}\right)\dot{\phi}_3^2 + \frac{\partial a_{11}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2$$
$$+ \left(\frac{\partial a_{12}}{\partial \phi_3} + \frac{\partial a_{13}}{\partial \phi_2} - \frac{\partial a_{23}}{\partial \phi_1}\right)\dot{\phi}_2\dot{\phi}_3 + \frac{\partial a_{11}}{\partial \phi_3}\dot{\phi}_1\dot{\phi}_3 = \Phi_1$$

with similar expressions for ϕ_2 and ϕ_3 coördinates.

To transform the coefficients to the form of velocity ratios, we note

$$\frac{\partial a_{12}}{\partial \phi_2} - \frac{1}{2}\frac{\partial a_{22}}{\partial \phi_1} = \sum M\left(K_{x\phi_1}\frac{\partial K_{x\phi_2}}{\partial \phi_2} + K_{y\phi_1}\frac{\partial K_{y\phi_2}}{\partial \phi_2}\right)$$
$$+ \sum I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_2}}{\partial \phi_2},$$

$$\frac{\partial a_{13}}{\partial \phi_3} - \frac{1}{2}\frac{\partial a_{33}}{\partial \phi_1} = \sum M\left(K_{x\phi_1}\frac{\partial K_{x\phi_3}}{\partial \phi_3} + K_{y\phi_1}\frac{\partial K_{y\phi_3}}{\partial \phi_3}\right)$$
$$+ \sum I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_3}}{\partial \phi_3},$$

and

$$\frac{\partial a_{12}}{\partial \phi_3} + \frac{\partial a_{13}}{\partial \phi_2} - \frac{\partial a_{23}}{\partial \phi_1} = \sum 2M\left(K_{x\phi_1}\frac{\partial K_{x\phi_2}}{\partial \phi_3} + K_{y\phi_1}\frac{\partial K_{y\phi_2}}{\partial \phi_3}\right)$$
$$+ \sum 2I K_{\psi\phi_1}\frac{\partial K_{\psi\phi_2}}{\partial \phi_3}, \quad \text{etc.}$$

Therefore, the generalized equation of motion in terms of partial velocity ratios, for three degrees of freedom for the coördinate ϕ_1 is

$$\sum M\Bigg\{K_{x\phi_1}\Bigg[K_{x\phi_1}\ddot{\phi}_1 + K_{x\phi_2}\ddot{\phi}_2 + K_{x\phi_3}\ddot{\phi}_3 + \frac{\partial K_{x\phi_1}}{\partial \phi_1}\dot{\phi}_1^2$$
$$+ \frac{\partial K_{x\phi_2}}{\partial \phi_2}\dot{\phi}_2^2 + \frac{\partial K_{x\phi_3}}{\partial \phi_3}\dot{\phi}_3^2 + 2\frac{\partial K_{x\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2 + 2\frac{\partial K_{x\phi_2}}{\partial \phi_3}\dot{\phi}_2\dot{\phi}_3$$
$$+ 2\frac{\partial K_{x\phi_3}}{\partial \phi_1}\dot{\phi}_3\dot{\phi}_1\Bigg] + K_{y\phi}\Bigg[K_{y\phi_1}\ddot{\phi}_1 + K_{y\phi_2}\ddot{\phi}_2 + K_{y\phi_3}\ddot{\phi}_3$$

$$
\begin{aligned}
&+ \frac{\partial K_{v\phi_1}}{\partial \phi_1}\dot{\phi}_1{}^2 + \frac{\partial K_{v\phi_2}}{\partial \phi_2}\dot{\phi}_2{}^2 + \frac{\partial K_{v\phi_3}}{\partial \phi_3}\dot{\phi}_3{}^2 + 2\frac{\partial K_{v\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2 \\
&+ 2\frac{\partial K_{v\phi_2}}{\partial \phi_3}\dot{\phi}_2\dot{\phi}_3 + 2\frac{\partial K_{v\phi_3}}{\partial \phi_1}\dot{\phi}_3\dot{\phi}_1\Bigg]\Bigg\} + \sum I\Bigg\{K_{\psi\phi_1}\Bigg[K_{\psi\phi_1}\ddot{\phi}_1 \\
&+ K_{\psi\phi_2}\ddot{\phi}_2 + K_{\psi\phi_3}\ddot{\phi}_3 + \frac{\partial K_{\psi\phi_1}}{\partial \phi_1}\dot{\phi}_1{}^2 + \frac{\partial K_{\psi\phi_2}}{\partial \phi_2}\dot{\phi}_2{}^2 + \frac{\partial K_{\psi\phi_3}}{\partial \phi_3}\dot{\phi}_3{}^2 \\
&\qquad + 2\frac{\partial K_{\psi\phi_1}}{\partial \phi_2}\dot{\phi}_1\dot{\phi}_2 + 2\frac{\partial K_{\psi\phi_2}}{\partial \phi_3}\dot{\phi}_2\dot{\phi}_3 + 2\frac{\partial K_{\psi\phi_3}}{\partial \phi_1}\dot{\phi}_3\dot{\phi}_1\Bigg]\Bigg\} = \Phi_1
\end{aligned}
$$

with similar equations for ϕ_2 and ϕ_3.

Evidently the terms in brackets are the generalized acceleration components and the multiplying partial velocity ratio reduces the generalized inertia loading to the particular coördinate under consideration. This latter is analogous to a mechanical varying gear ratio with respect to any particular coördinate, for any given inertia element of the mechanism.

CONCLUSION.

The subject of dynamics of machinery offers a particular theoretical field for the application of general dynamical methods in a category quite comparable to gyrostatics and similar provinces in dynamics. Considering its practical importance it is fundamental in the field of technical engineering and offers a very unique field in applied physics.

A primary object of these articles has been to maintain only the theoretical aspect of the subject as a branch of general dynamics. The development in this work of the dynamical equations directly in terms of velocity ratios and their derivatives for both one and several degrees of freedom is of particular use in the specialized subject of dynamics of machinery. It gives a connecting link between the now separate but well developed field of engineering kinematics and the dynamics of mechanisms. The preliminary analysis, however, offers considerable extension in both analytical and graphical procedures. Emphasis has also been made on the use of redundant coördinates with the equations of conditions or constrained velocity relations as a procedure of determining constraint reactions and internal reactions in a mechanism.

Other branches of this subject that await considerable development are the application of gyroscopic theory in machinery, mechanical devices, and control apparatus. Also, an important phase is in connection with the oscillations of mechanical electromagnetic systems in control apparatus and with prime movers, synchronous motors, etc., in complex electric power systems. The field of vibration of machinery is under way, but it will always have a fertile field of many applications and further extension in theory.

Very closely connected with the subject and really an integral part pertains to the study of electromagnet reactions and fluid dynamics.

These articles will quite fulfill their purpose if the presentation of the subject adds some further interest to the possibilities of the subject of dynamics of machinery as a branch of Applied Physics and Technical Engineering.

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The presentation is an outcome of a proposed dissertation subject by the late Dr. A. G. Webster to whom the author is greatly indebted for much inspiration on the subject of dynamics and its theoretical applications. Particular acknowledgment is due Dr. R. H. Goddard and Dr. A. W. Duff and also Dr. E. B. Wilson and Dr. C. A. Adams for their criticisms and review of this work in its nucleus form as a portion of a Dissertation Subject. To the late Prof. G. Lanza and in particular to Dr. S. Timoshenko, to Dr. H. N. Davis, Dr. E. V. Huntington, Prof. C. E. Fuller, Dr. H. C. Richards and to Prof. J. C. Riley, the author is also indebted in connection with the discussion, advices, and suggestions on dynamical problems intimately connected with this work. Particular acknowledgment is also due Dr. Howard McClenahan for his interest and for his critical review of the manuscript as a subject for publication as well as for several fundamental suggestions in the presentation of the work. Thanks are also extended to Mr. A. Rigling in connection with the editorial work.

Acknowledgment is due Mr. A. Madle for permission to consider the Relay Motor Oscillating axle for illustrating the use of velocity ratios in conjunction with a complex dynamical problem, and Mr. B. H. Slocum for suggestions as to an actual problem on the dynamics of a gear train on a rotating frame.

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NOTES FROM U. S. BUREAU OF STANDARDS.*

LIQUEFACTION OF HELIUM.

On April 4, Dr. George K. Burgess, director of the National Bureau of Standards, announced that helium had been liquefied in the bureau's cryogenic laboratory on the afternoon of Friday, April 3. Liquid helium temperatures were maintained for two hours and a temperature of – 271.2° C. (– 456° F.) was reached, which is only 1.9° C. (3.4° F.) above Absolute zero, – 273.1° C. (– 459.4° F.). The temperature of liquid helium at atmospheric pressure is – 269° C. (– 452° F.), but it can be cooled further by reducing the pressure of the helium vapors over the boiling liquid with a vacuum pump. As a result of many months of work in the cryogenic laboratory not only was the liquefaction of helium accomplished for the first time in the United States but also the lowest temperatures ever attained in this country were reached.

The helium liquefier was operated for the first time on March 18, at which time it is believed liquid helium was produced. The positive evidence of the presence of liquid helium obtained April 3 was not obtained March 18, since the apparatus with which it was proved that liquid helium was being produced was added after the experiment of March 18; however, no changes had been made in the liquefier. The successful production of liquid helium will permit the bureau to extend its investigations of the properties of matter to the lowest attainable temperatures. It may also assist in the establishment of a new low fixed point on the International Temperature Scale, the establishment of these natural constants being one of the primary functions of the bureau.

The initial step in the process of liquefaction of helium is the purification of the gas to remove substances which would otherwise freeze in the liquefier and stop the flow, since at liquid helium temperatures, all substances with the single exception of helium are solids. After purification the gas enters the liquefier which is inclosed in a metal case surrounded, except for an opening at the top, with liquid air at a temperature of − 190° C. (− 310° F.). In the liquefier the helium expands through a valve from a high pressure to a low pressure. The helium in expanding becomes colder and the refrigeration effect is used to lower the temperature of the compressed helium that follows it. The temperature continues to fall until the liquefaction point is reached: − 269° C. (− 452° F.). It is necessary to reduce the temperature of the compressed helium before expansion, with solid or liquid hydrogen, as otherwise the helium would become warmer instead of colder. Liquid hydrogen boils at −253° C. (− 423° F.) and freezes at −259° C. (− 434° F.). In order to liquefy helium, it was necessary therefore to make first liquid air and liquid hydrogen. In the liquefier the helium flows through a series of coils of metal tubing suspended inside a succession of metal cases and vacuum jacketed vessels so arranged as to prevent as far as possible any transfer of heat to the coldest parts of the liquefier from a warmer exterior. The amount of refrigeration that is generated is so small that the success of the apparatus depends on obtaining almost perfect insulation.

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* Communicated by the Director.

Proof of the presence of liquid helium was obtained by two independent methods. The temperatures were measured with a vapor pressure thermometer filled with helium, the bulb of which was immersed in the receiver of the liquefier in which the liquid helium collected. The bulb was connected to a mercury manometer outside the liquefier. The vapor pressure of the liquid helium in the bulb was determined by measuring the length of a column of mercury which the vapor would support. Using a known relation connecting the vapor pressure of helium with its temperature, the temperatures of the liquid helium in the liquefier were determined. The vapor pressure thermometer was so made that the volume of the vapor space could be changed by raising or lowering the levels of mercury columns in a large glass bulb and in an auxiliary glass tube. The fact that the vapor pressure, as read on the mercury manometer, did not depend upon the volume of the vapor space was proof of the presence of liquid helium. In order to obtain still further evidence, a lead and tin electrical resistance unit was constructed and placed in the liquid helium reservoir beside the bulb of the vapor pressure thermometer. It is already known that lead loses all of its electrical resistance and becomes a perfect conductor at − 266° C. (− 448° F.) and that the same is true for tin at − 269° C. (− 453° F.). This is called supraconductivity. That these temperatures were attained was therefore proved by the fact that both the tin and the lead lost their resistance and became supraconducting.

This phenomenon of supraconductivity is only one of the strange effects of these very low temperatures. Although tin, lead, mercury, and a number of other metallic elements and alloys become perfect conductors, metals like copper, gold, and silver, which at ordinary temperatures are the best conductors, do not become supraconducting, so that the metals which at ordinary temperatures are considered poorer conductors become infinitely better conductors at liquid helium temperatures than copper, gold, and silver.

Helium has been liquefied before in three other countries, first in Holland at the University of Leiden, next in Canada at the University of Toronto, and then in Germany at the Reichsanstalt and at the University of Berlin. The liquefier constructed by the Bureau of Standards is essentially like the one designed by Dr. Martin Ruhemann and used by him to make liquid helium at the University of Berlin. Some changes in the nature of improvements have been made.

SUNLIGHT AND DAYLIGHT FILTERS.

Miscellaneous Publication No. 114 of the Bureau of Standards which was released last month, deals with filters for the reproduction of sunlight and daylight, and the determination of color temperature. For scientific and technical use in photographic sensitometry, colorimetry, and photometry, the following filters, reproducible from specification, have been developed:

(1) A series of filters by means of which the spectral energy distribution of a Planckian radiator (black body) at any temperature from 2,300 to 4,000° K, inclusive, may be

converted into a close approximation of the spectral energy distribution of mean noon sunlight at Washington. The data cover the range from 350 to 720 millimicrons. Light transmissions have been computed for all the filters.

(2) By designing these filters so that they all give an exact color match with the adopted mean sunlight, in addition to a close energy match, they may be used to calibrate the intensity and color temperature of incandescent lamps at any color temperature above 2,300° K, provided a source is available calibrated for intensity and color temperature at some one color temperature within this range. The comparison of intensities of incandescent lamps at different color temperatures may thus be made under conditions of exact color match.

(3) A second series of filters has been developed for converting the color of a Planckian radiator at 2,848° K to that of any Planckian radiator between 3,500 and 10,000° K, inclusive.

(4) Seven additional filters, not in the above series, are also described, viz.:

(*a*) A filter for converting 2,848° K to the color of the sun outside of the earth's atmosphere, which satisfactorily represents the color of overcast sky, this filter thus affording a reproducible daylight standard. By combining this filter with sources at various color temperatures between 2,300 and 3,000° K, colors closely matching various phases of sunlight and daylight may be obtained.

(*b*) A filter for converting 2,848° K to Abbot-Priest sunlight.

(*c*) A filter for converting acetylene to mean noon sunlight at Washington.

(*d*) and (*e*) Two filters, one for converting 2,848° K to noon sunlight at the summer solstice at Washington and the other for converting 2,848° K to noon sunlight at the winter solstice at Washington.

(*f*) and (*g*) Two filters designed to cover a wide range of black-body color temperatures by varying the color temperature of the light source between the limits of 2,000 and 3,100° K. With this variation one of the filters covers the black-body scale from 2,650 to 5,000° K and the other from 4,000 to 19,000° K.

The filters consist of a two-compartment cell with three borosilicate crown glass windows, the two compartments being filled respectively with solutions *A* and *B*, having compositions as follows:

Solution A.

Copper Sulphate ($CuSO_4 . 5H_2O$)	c grams
Mannite ($C_6H_8(OH)_6$)	c grams
Pyridine (C_5H_5N)	30.0 cc.
Water (distilled) to make	1000.0 cc.

Solution B.

Copper Sulphate ($CuSO_4 . 5H_2O$)	c_1 grams
Cobalt Ammonium Sulphate ($CoSO_4 . (NH_4)_2SO_4 . 6H_2O$)	c_2 grams
Sulphuric Acid (sp. gr., 1.835)	10.0 cc.
Water (distilled) to make	1000.0 cc.

The values of c, c_1, and c_2 vary from one filter to another.

The design of the filters has been based upon extensive spectrophotometric measurements of the component solutions and cell, including a study of Beer's law over the ranges of concentrations used, and upon methods of computation giving accurate color matches and relatively good spectral energy matches between the source-and-filter combination and the energy distribution which it was sought to duplicate. Detailed studies have been made of the reliability of the data, the spectrophotometric reproducibility of the chemicals, the permanence of the solutions, their temperature coefficients, and various other factors of importance in the preparation and use of the filters.

The paper contains 33 figures and 26 tables, giving among other things the energy distributions and fundamental spectrophotometric and colorimetric data used in the computations, auxiliary spectrophotometric and colorimetric information obtained during the investigation, and the trilinear coördinates of the various source-and-filter combinations and of the Planckian radiator from 1,600 to 20,000° K computed on the basis of the adopted mean sun as the "neutral" stimulus. The filters are further described in 38 charts, each chart giving the chemical formula, the spectral transmission, and the light transmission of the filter, as well as the spectral energy distribution of the source, that of the source and filter combined, and the energy distribution which serves as the ideal in that particular case.

Copies of this publication may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 45 cents each.

METHODS AND APPARATUS USED IN SPECTROPHOTOMETRY.

In a paper which has been prepared for the Journal of the Optical Society of America, brief descriptions are given of the methods and apparatus used in making spectrophotometric measurements in the colorimetry section of the bureau. Full descriptions have been given in the publications to which reference is made. What is noted particularly in the present paper are those features of the methods which tend to insure reliability in the values obtained. It is shown that the photometric scale of the Koenig-Martens spectrophotometer can be independently checked and greatly extended by means of the rotating sectors. The means taken to eliminate wave-length, slit-width, and stray-light errors are noted, particularly the use of homogeneous light of certain wave-lengths in the measurement of transmission.

The degree of agreement ordinarily obtained with the visual, photoelectric, and thermoelectric methods is shown by several illustrations. The reliability of the non-visual methods is checked by their agreement with the visual method at those wave-lengths where the latter is reliable. In turn, the non-visual methods serve to maintain the reliability of the measurements throughout the end regions of the visible spectrum where the sensibility of the eye is extremely low.

TWO NEW CRYSTALLINE DIFRUCTOSE ANHYDRIDES FROM HYDROLYZED INULIN.

It has been shown by Jackson and Goergen (Bureau of Standards Journal of Research, Vol. III, No. 1; July, 1929; RP–79) that inulin upon hydrolysis in aqueous solution with sulphuric acid yields a mixture of sugars consisting of 92 per cent. fructose, 3 per cent. glucose, and 5 per cent. of a group of nonreducing disaccharides, one of which was isolated in crystalline form and shown to be a difructose anhydride having a specific rotation of + 27.0° for sodium light at 20° C.

From the mother liquors which were separated from the first difructose there was obtained a second crystalline substance which by elementary analysis and determination of molecular weight was shown to be a disaccharide of empirical formula $C_{12}H_{20}O_{10}$. It yielded upon hydrolysis with 0.2 normal sulphuric acid for 100 minutes at 100° C. two molecules of fructose. It was therefore designated Difructose Anhydride II. After careful purification it showed a rotation of + 13.85 and a melting point of 198° C.

The mother liquors from which the second difructose had crystallized soon deposited a third crystalline substance which similarly showed an empirical formula of $C_{12}H_{20}O_{10}$. When treated with 0.2 normal sulphuric acid at 100° C. for 110 minutes it was partially hydrolyzed to a levoratory substance which by chemical analysis was found to consist solely of fructose. It was therefore designated Difructose Anhydride III. The polariscopic analysis of the hydrolyzed sugar indicated the formation of an intermediate product of hydrolysis.

The purified substance showed a specific rotation of + 135.64 and a melting point of 162° C.

From the 5 per cent. nonreducing portion of hydrolyzed inulin have now been isolated the three difructose anhydrides whose properties are summarized in the following table:

Property Studied.	Difructose Anhydride.		
	I.	II.	III.
Specific rotation (water)	+ 27.0	+ 13.85	+ 135.64
Melting point, ° C.	164	198	162
Crystalline form	Thin plates.	Truncated prisms.	Bipyramids.
Molecular weight (Calculated, 324)	307	310	308
Approximate solubility in water *	32	32	38

* Expressed as grams of sugar in 100 grams of solution at 20° C.

The three sugars resemble each other in respect to their great resistance to hydrolysis and their comparatively slight solubility in water. They differ in rotatory power, crystalline structure, and melting point.

The mean rotation of the three sugars is + 58.8 which is in essential agreement with the rotation (+ 55) of the nonreducing portion of hydrolyzed inulin as first isolated by Jackson and Goergen. This agreement indicates that the 5 per cent. nonreducing substances consist solely of these three sugars occurring in equal proportions.

The results of this investigation corroborate the conclusions drawn in a previous article (R. P. 251) in which it was shown by arithmetical reasoning that a 5.2 per cent. nonreducing portion of hydrolyzed inulin which consisted of three disaccharides occurring in equal proportions predicated a molecular weight of inulin of approximately 18,000.

A complete report of this work will appear in the April number of the Bureau of Standards Journal of Research.

PURITY OF REAGENT CHEMICALS.

At the recent meeting of the American Chemical Society in Indianapolis, Edward Wichers of the Bureau of Standards presented a paper summarizing the results of tests of over 200 reagent chemicals purchased for use in the bureau's laboratories during the last 2½ years. About three-fourths of chemicals tested, representing a considerable variety of materials whose purity is of much importance to the analytical chemist, were purchased under the quality specifications prepared by the Society. Only two-thirds of the group of chemicals were found to conform entirely to the specified requirements.

Of the second group of chemicals, which were purchased under the general requirement of "analytical reagent quality," only 60 per cent. were found to conform fully to the standards indicated by the labels on the containers. Considerable variations were found in the success of the different producers of these materials in meeting the requirements of their own standards or of the American Chemical Society specifications.

The paper included a discussion of a number of reagents of especially inferior quality, and pointed out means by which improvement can be made in the quality of reagent chemicals in general.

THE VIBRATION OF U BARS.

An item under this title appeared on page 26 of Technical News Bulletin No. 167 (March, 1931). A typographical error occurs in the third paragraph. Beginning with line five the sentence should read: "In this connection a new proof is given of Rayleigh's method for determining the fundamental mode of vibration."

TURBULENCE IN WIND TUNNELS.

One of the questions of current interest in aerodynamics is the desirability of having a large or a small turbulence in wind tunnels for the routine testing of airplane models. Based on a long series of experiments, the bureau has concluded that the wind tunnel should be so designed as to have small turbulence, because increased turbulence does not reproduce accurately the effect of increased scale of the model and because it is easier to introduce turbulence, if desired, than to remove it. The experiments in question were made in the bureau's 54 inch wind tunnel, in which various methods for reducing the turbulence were tried. Changes in the size and shape of the honeycomb cells had very little effect. The distance between the honeycomb and the measuring station was found to be the variable of primary importance, an increased distance resulting in a lower turbulence. An additional reduction was secured by using a large area-reduction in the entrance cone, with the honeycomb at the extreme entrance end. It was not found practical to operate the tunnel without any honeycomb.

This work was conducted with the coöperation and financial assistance of the National Advisory Committee for Aeronautics. The measurement of turbulence was made by the use of spheres and by the hot wire anemometer described in National Advisory Committee for Aeronautics Technical Report No. 342.

WIND PRESSURE ON A MODEL OF A MILL BUILDING.

In continuation of a program of investigation of wind pressure on structures, measurements have been made in the bureau's 10-foot wind tunnel on the distribution of pressure over a model of a mill building, both with and without a monitor ventilator. Speeds up to about 55 miles per hour were used, and there was little evidence of scale effect. The effect of the ground on which an actual building stands was simulated by a platform on which the model was mounted. Measurements were made for three wind directions, one normal to a side wall, one normal to an end wall, and one at 45°.

The maximum average loads were obtained with the wind at 45°, but in many cases fairly large areas were subjected to loads as great as twice the average load on the whole face. The loads on the roof of the monitor were greater than those on the roof of the main building, and in both cases the load was in such a direction as to lift the roof. The greater part of the model was subject to pressures lower than the static pressure.

The detailed results cannot well be abstracted. The lowest pressure was below the static pressure by 3.3 times the velocity pressure, corresponding to a local loading of 47 pounds per square foot at a true speed of 75 miles per hour. This high loading was confined to an extremely small area on the roof of the monitor. Fairly large areas were subject to upward loads corresponding to 28 pounds per square foot at a true speed of 75 miles per hour. These values are computed on the assumption that the pressure within the building equals the static pressure, and therefore will be modified by the presence of open windows or other openings. The loads may be greater or less than those given, depending on the location of the openings and the direction of the wind.

The complete report of this investigation will be published in the April number of the Bureau of Standards Journal of Research.

PROGRESS IN AERONAUTIC RADIO RESEARCH.

The bureau has made performance tests of its recently developed simultaneous radio telephone and range-beacon transmitter. This development permits an airplane pilot to receive simultaneously voice messages and visual radio beacon signals, all on a single receiving set. Tests made at a distance with a commercial aircraft receiving set showed that the quality of the received speech signals is substantially better than that obtained in existing airplane radiotelephone reception, and that the operation of the beacon course indicator is equivalent to that obtained with existing visual type range-beacons. Test flights were made to determine the useful distance range of the combined transmitter in its present adjustment. At a distance of 100 miles from the transmitting station, it was still unnecessary to employ the full sensitivity of the receiving set. Increasing the effective height of the transmitting antenna system will result in still greater useful distance range. These results insure that simultaneous transmission of the broadcast and range-beacon services and the simultaneous reception of these services on the airplane, are entirely practicable for use on the airways.

A brief description of a radio system under development by the bureau as an aid to the prevention of collision between aircraft in flight under conditions of poor visibility was given on page 117 of Technical News Bulletin No. 164 (December, 1930). This system involves transmission of a warning signal on a very high frequency from each airplane and reception on the same frequency on each airplane. Upon receiving the warning signal from an airplane ahead of him, the pilot will maneuver his plane to reduce the intensity of received signal, thereby automatically deviating from the direction of danger. On account of the very high frequency and relatively short distance required, the power needed is very low. A transmitting set for this purpose has been constructed employing a single commercial receiving tube and operating on a frequency of 165,000 kilocycles. The receiving set also uses a single receiving tube of the same type in a very stable super-regenerative circuit. To secure visual indication of the received signal, a copper-oxide rectifier and microammeter were connected to the output of the receiving set. A directional receiving antenna system was used, consisting of three doublet antennas approximately 3 feet long and spaced 18 inches apart, one being connected to the input of the receiving set. In preliminary tests a useful distance range of 3 miles was obtained, which is about the distance over which it would be desirable to have the collision-prevention system work. The apparatus does not yet have sufficient stability for practical application; the experiments continue.

An indicator has been developed at the bureau for the visual type radio range-beacon which may have some advantages over the reed indicator regularly employed. In the new instrument a pointer remains at a zero mark when the airplane is on the course and swings to right or left as the airplane deviates from the course. Numerous flight tests have been made of this device and its practicability demonstrated. One of these instruments has been loaned to an air transport operator for service tests in regular operation flights in connection with a radio range-beacon of the visual type installed on the airways. Until such tests are completed, it cannot be stated whether this instrument is preferable to the reed indicator. The device is operated by a reed converter, i.e., a reed indicator in which, besides the set of driving coils for operating each reed from the received beacon signals, a set of pickup coils is provided for each reed. The motions of the two reeds induce alternating voltages in the two pickup coils, which are proportional to the reed vibration amplitudes and consequently to the intensities of the two low frequencies constituting the received beacon signal. The induced alternating voltages are rectified by means of copper-oxide rectifiers and the rectified voltages applied in opposition to the terminals of the zero-center instrument which the pilot sees. One of the problems which had to be considered in connection with this instrument was the possibility that the beacon signals might stop, or the receiving set fail, and the instrument continuing to read zero, the pilot would be deceived into thinking his course was still being indicated. To preclude this possibility, a volume indicator is employed. A paper describing the new instrument is in preparation. When issued, a notice will be given in these columns.

PIPE LINE CURRENTS AND SOIL RESISTIVITY.

The present tendency in the protection of pipe lines against soil corrosion is away from a uniform coating for the entire length of the line and toward the application of coatings selected with respect to the corrosive character of the soils involved. This calls for a knowledge of varying soil conditions and a means of locating local corrosive soil areas. Such areas, commonly termed "hot spots," are prevalent in many types of soils and it is in these regions that pipe lines suffer the greatest deterioration. The location of such corrosive areas is a matter of considerable importance, both with respect to the selection of coatings for new pipe lines and the reconditioning of old ones.

An extensive investigation of a dozen pipe lines ranging from the gulf coast to southern Kansas revealed an apparent correlation between pipe line currents, soil resistivity, and corrosion. Galvanic currents of measurable magnitudes were found to be flowing on all pipe lines examined. As a rule, lines were found to be collecting current in areas of normal and high soil resistivity and losing current in areas of low resistivity. Many cases of abrupt loss or discharge of current occurred in soils of unusually low resistivity. In such areas the pipe lines were found to be badly corroded.

Although no direct relation was found to exist between electrical resistivity of soils and their corrosiveness, abrupt changes in resistivity and unusually low resistivity were found to be significant with respect to corrosion. Soils having a resistivity of about 500 ohm cm. or less were invariably found to be highly corrosive. A better relation between resistivity and corrosiveness exists in alkali than in acid soils.

To expedite the measurement of soil resistivity, a simple piece of apparatus was developed, consisting of two metal tipped oak rods with a milliammeter mounted on one and a small flashlight battery on the other. By measuring the apparent resistance between the metal terminals, the approximate resistivity of the soil can be calculated if the constant of the apparatus is known. Since polarization is largely a cathode effect, it can be partly eliminated by making the cathode terminal much larger than the anode, thereby reducing the current density on it. A low resistance meter, such as a milliammeter, is found to be preferable to a voltmeter as with the former practically the full voltage of the test battery is applied between the electrodes, irrespective of the meter reading, while with the latter a large part of the battery voltage is consumed within the meter itself.

This apparatus permits of making frequent and rapid tests of earth resistivity along a pipe line or along a projected pipe line. Although it is not claimed that absolute values of resistivity are obtained with this apparatus, the results are sufficiently accurate for all practical purposes provided the soil is not too dry. It is important that earth be tested at the pipe level and where there is sufficient moisture to afford nearly maximum conductivity. Damp soil which will pack in the hand will usually show conductivity approaching that in a saturated condition.

The technique of surveying pipe lines for galvanic currents and other features of the investigation will be discussed in the April number of the Bureau of Standards Journal of Research.

THERMAL EXPANSION OF SOME SODA-LIME GLASSES.

In Technical News Bulletin No. 163 (November, 1930) a statement was published regarding a method for computing the thermal expansion of soda-lime glasses from room temperature to the "critical temperature."

Further study of the data obtained has led to two new equations from which the expansion of the glasses considered can be computed with considerable accuracy in the temperature ranges 25 to 100° C. and 100 to 400° C.

The expressions are:

$$E_{25^\circ-100^\circ} = 0.004P_1 + 0.325P_2 + 0.125P_3 - 0.00186\,(P_2 + P_3),^2$$

$$E_{100^\circ-400^\circ} = 0.015P_1 + 1.655P_2 + 0.655P_3 + 0.0045\,(P_2 + P_3)^2 - 0.6,$$

in which E is the increase in length expressed in microns per centimeter between the temperatures indicated, and P_1, P_2 and P_3 are the percentages of silica, soda, and lime respectively. Further studies of the expansion data are being made to see if the "critical temperatures" and the initial softening points can be computed.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, APRIL 15, 1931.

The monthly meeting of The Franklin Institute was called to order at eight-fifteen by Mr. Nathan Hayward, President.

He at once called upon the Secretary for the business of the evening. The latter reported that the minutes of the March meeting had been published in full in the Journal of the Institute for April and moved that they be approved as printed. The motion was seconded and unanimously adopted.

The Secretary then stated that since the last report concerning the membership of the Institute, the following changes had been made: Three new Resident Members and four new Non-Resident Members had been elected. Two members had died during that period.

The only other business consisted of various notices of functions to occur in the early future in which the members of the Institute might be interested.

The Secretary mentioned particularly the Medal Day exercises of the Institute to be held on the afternoon of May twentieth, and urged the members to attend if possible to do so.

The President then called upon the speaker of the evening, Dr. W. F. G. Swann, at eight-twenty-four, who presented "The Annual Report of the Work of the Bartol Research Foundation." Dr. Swann presented in graphic fashion an outline of the various important pieces of research which are being carried on in the Bartol Laboratories. The report will be published in full in an early number of the Journal.

Dr. Swann's talk was followed by a considerable amount of animated discussion.

There being no further business, the President declared the meeting adjourned, with an expression of thanks to Dr. Swann, at ten o'clock.

Howard McClenahan,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, April 8, 1931.*)

Hall of the Committee,
Philadelphia, April 8, 1931.

Mr. Lionel F. Levy *in the Chair.*

The following reports were presented for final action:

No. 2938: Walton Clark Medal.

This report recommended the award of the Walton Clark Medal to Mr. Daniel J. Young, of Tacoma, Washington, "In recognition of his outstanding contributions to the manufactured gas industry as embodied in his energy-conserving 'Back-Run Process.'"

No. 2939: George R. Henderson Medal.

This report recommended the award of the George R. Henderson Medal to Doctor Arthur Newell Talbot, of Urbana, Illinois, "In consideration of his practical development of the Railway Transition Spiral and for his creative guidance of American Railway Engineering Association's tests on structural and track materials for railway building and maintenance which resulted in a wide extension of the knowledge of such materials and in the promotion of safety in railroading."

Report on Protest:

Case No. 2883: Monroe Calculating Machine.

Original award recommended by Sub-Committee was sustained, and recommendation is to be readvertised.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, April 15, 1931.*)

RESIDENT.

MR. EDWARDS KNEASS, Engineer, I. T. E. Circuit Breaker Company, 501 North Nineteenth Street, Philadelphia, Pennsylvania. For mailing: 2225 Pine Street, Philadelphia, Pennsylvania.

NON-RESIDENT.

MR. ERNEST A. FAIRMAN, Silviculturist, Soc. Anon Gan. y Com., Chile, South America. For mailing: Casilla, C., Magallanes Chile, South America.

MR. PATRICK F. FOLEY, Chief Inspector of Elevators, Municipal Building, Room 2000, Borough of Manhattan, New York. For mailing: 917 Seventy-seventh street Brooklyn, New York.

MR. WILLIAM HOWARD MCCOY, Manager of General Motors Research Experimental Production Machine Shop, Detroit, Michigan. For mailing: 17593 Stoepel, Detroit, Michigan.

CHANGES OF ADDRESS.

BRIGADIER GENERAL W. W. ATTERBURY, Room 1932, Broad Street Station Building, Philadelphia, Pennsylvania.

DR. LESLIE R. BACON, 314 Highland Avenue, Lansdowne, Pennsylvania.

MR. NATHANAEL G. HERRESHOFF, 6 Walley Street, Bristol, R. I.

MR. DONALD L. KELLOGG, Armstrong Cork Company, Pittsburgh, Pennsylvania.

LIEUTENANT COLONEL CHESTER LICHTENBERG, General Electric Company, 1635 Broadway, Fort Wayne, Indiana.

MR. WILLIS A. NAUDAIN, Post Office Box 846, Wilmington, Delaware.

COLONEL SAMUEL REBER, 570 Lexington Avenue, New York City, New York.

NECROLOGY.

Mr. A. Markle, Hazleton, Pennsylvania.

LIBRARY NOTES.

RECENT ADDITIONS.

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THE NAIRNE WAY-WISER.

The Institute has recently come into possession of a device for measuring distances between given points. The instrument, which is more than a century and a half old, was presented by Mr. Harrold E. Gillingham, of Philadelphia, who has furnished the following descriptive note:

A SURVEYOR'S MEASURING WHEEL OR WAY-WISER.

AN INSTRUMENT FOR MEASURING SHORT DISTANCES ON THE ROAD OR OTHER LAND SURFACES. THE ANCESTOR OF THE CYCLOMETER AND TAXIMETER.

BY

HARROLD E. GILLINGHAM.

We have grown so accustomed to many inventions of these modern times and are inclined to take so much for granted that the average person gives little heed to the past, and seldom thinks of the need our forebears must have felt for distance-measuring devices. Comparatively, they may have had just as much reason as we for wanting to know the correct distances traveled when they went a journeying.

One reads that milestones erected along the Roman *Via Publica* in the time of Publius Popillius Laenas (consul in 132 B.C.), bore inscriptions to commemorate the builder of the highway, and yet we give little heed to the methods used to accurately measure the distances as indicated by milestones along the roads of to-day.

Many are well aware of the surveyor's chain or tape [1] for accurately measuring short distances, and it is readily conceivable how burdensome this method would become if used to set milestones. It naturally became necessary to provide some instrument for greater distances, and the *Way-Wiser*, or Odometer, was the result of such experimentation.

In 1528 Jean Francois Fernel (1497–1558), a French physician and mathematical student, measured the distance from Paris to Amiens by the revolutions of his carriage wheel. He no doubt adapted the principle of the Greek hodometer (or odometer) to the wheels, by an arrangement of cogwheels and endless screws on the same axes, for this purpose. John Evelyn, the English diarist (1620–1706) and Secretary of the Royal Society, mentions a similar apparatus in his diary notes on August 6, 1656, when he records the application of a *Way-Wiser* to the wheels of a coach, which measured the miles.

The surveyor's measuring wheel, sometimes called a Perambulator, an Odometer or Way-Wiser, was the natural result of the earlier instruments when shorter distances, easily covered by a pedestrian, were required. It might well be called the great-great-grandfather of the modern cyclometer, taximeter or speedometer. The term Odometer is usually applied to a measuring instrument affixed to a wheel of a vehicle; while a perambulating instrument was called a Way-Wiser.

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[1] Dr. R. T. Gunther, in *Early Science at Oxford* (Vol. 1, p. 372), illustrates a surveyor's chain, (*c*) 1590, described by Cyprian Lucar in *A Treatise named Lucarsolace*.

FIG. 1.

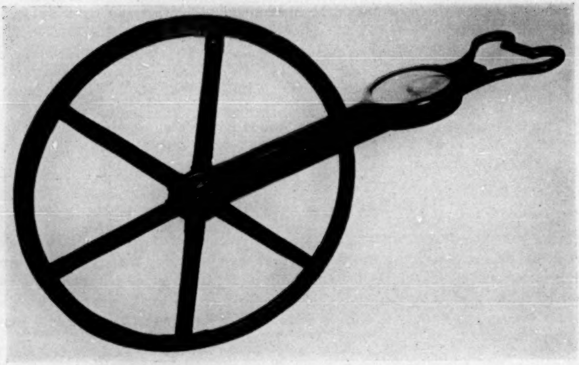
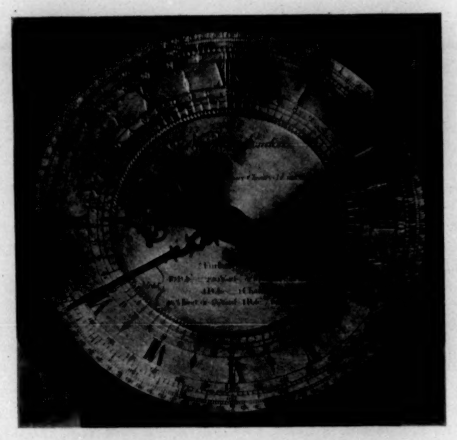


FIG. 2.



Just who first invented or produced this form of perambulating measuring wheel is uncertain. It is assumable they were made by the various mathematical instrument makers of the 17th and 18th centuries. The Way-Wiser herein mentioned was produced in the workshop of Edward Nairne of 20 Cornhill, London, working in that city as early as 1761, and who made telescopes, sun-dials and other mathematical instruments. The steel-tired mahogany wheel is 31½ inches in diameter, and from the axle to the end of the mahogany handle frame it measures 38 inches. The brass dial on centre of frame is 7½ inches across. On the dial plate are five circular graduated scales of measurement: the outer one for feet, chains and links; the next is for yards, the third for poles; the fourth and wider circle, with Roman letters, indicates the miles; while the fifth or inner ring denotes the furlongs. Within the above—so that "he who runs may read"—is engraved in script—

NAIRNE, LONDON.

The outer circle contains Chains & Links.

Note
8 Furlongs = 1 Mile.
40 Poles = 220 Yards, or 10 Chains, 1 Furlong.
4 Poles = 1 Chain.
IN 92
16½ Feet, or 5½ Yards, 1 Pole: 7. 100. 1 Link.

The English antique dealer from whom this instrument was secured bought it at a public sale held at Flaxley Abbey, near Newnham, Gloucestershire. The history of Flaxley Abbey goes back to the 12th century, and it was the reputed home of Sir Roger de Coverly's *Widow Boevey.*

This identical piece may have been used by the *Way-Wardens*, or supervisors of the roads of that section of England, a century and a half ago.

There are many persons living today in England who recall the use of such instruments to settle disputes as to the size of a field or the distance driven by a London cabman. Imagine the consternation of the passers-by to see an officer pushing such a thing along the highway, if one protested an overcharge by the taxi driver!

The Franklin Institute has on display a perambulating Way-Wiser or Odometer, made by Heller and Brightly, makers of mathematical instruments in Philadelphia from 1870 to 1884. Two other such instruments, of earlier make than the above, are exhibited in the Old Ashmolean Museum of Oxford, England. One of these is by Cary of London (1799–1830), and loaned by Capt. Spencer Churchill; the other, having six dials for various distances, is the work of Johan Anton Wisenpainter of Eichstadt. A Wilkins Odometer, (*c*) 1678, is also shown. Other European museums have on display Hodometers, or wagon wheel distance measuring instruments, the earliest of which can be seen in the *Mathematisch-Physikalischer Salon* of the Dresden Museum. One by Thomas Rückert of Augsburg, 1575; another by Feyhel, dated 1580, and the third by Christoff Tressler (or Treschler), made in 1584.

BOOK REVIEWS.

The Stars in Their Courses. By Sir James Jeans, M.A., D.Sc., Sc.D., LL.D., F.R.S. x–173 pages, xlvi plates and 2 star maps, 12mo. New York, The Macmillan Co., 1930. Price $2.50.

To members of The Franklin Institute, a book by Sir James Jeans is particularly timely. Shortly after this issue appears, Sir James will speak at the Institute when he, together with Dr. Willis R. Whitney, is awarded the Franklin Medal. Doubtless his address will deal with some of the subjects covered in this, and other of his recent books, such as "The Universe Around Us," "The Mysterious Universe," and the more technical "Astronomy and Cosmogony."

Of the four books, "The Stars in Their Courses" is decidedly the most elementary. The first two were really masterpieces of popular writing, and as simple as possible with such subjects. Though written with all the delightful style for which the author is distinguished, it is true that in parts they were rather difficult for the lay reader. Every part of the present book, on the other hand, can be comprehended by an intelligent person with no scientific training at all. Yet there is no feeling that he is "writing down" to the layman, a blemish that so often mars a popular scientific book. Even to the astronomer, or to the scientist familiar with the subject, this work should be of intense interest. Perhaps such a person might learn nothing new from its pages, though that seems most unlikely, but even so the author's mode of expression should afford some good ideas.

For instance, when he says, "Leave only three wasps alive in the whole of Europe and the air of Europe will still be more crowded with wasps than space is with stars," he gives us a far more vivid picture of the emptiness of space than he would with any numerical statements of distances between stars. Or when he points out that the stars show "as great a range of candle-power as there is between a glow-worm and a searchlight," the reader gets a good idea of the variation in stellar brightnesses.

The book had its origin in a series of radio talks given through the British Broadcasting Company, the government sponsored corporation that has a monopoly on broadcasting in Great Britain. Whatever may be the evils of government operation, one who has had experience with the radio here, and the "wireless" in Britain, sometimes feels envious, and wishes that American broadcasters were under some similar control. Certainly nothing like these talks on astronomy has ever been within reach of American listeners, and this is not because we have no competent speakers.

In preparing the talks for publication, Sir James has expanded them to about twice their original length, but he has preserved the easy and familiar style which the microphone, like the popular scientific work, demands. In less than 200 pages he touches on practically every phase of modern astronomy. The numerous half-tone illustrations are excellently reproduced from some of the finest astronomical photographs, several of which have never before been published.

Starting out with the starry skies themselves, he discusses the motions of the stars and planets. Then he takes his readers on an imaginary journey around the Solar System, past the Moon and into the Sun itself. This permits him to treat of modern theories of stellar structure, and also of atomic structure. Then follows a journey in time, back to the birth of the Solar System, probably the result of the close approach of another star to the Sun. He next describes the outer planets, including Pluto, and passes out to the stars and thence into the depths of space. He concludes with a chapter on "The Great Universe," which is apparently expanding like a soap bubble while being blown. This is the view of the Belgian priest, Father Lemaître, to which Sir James seems to subscribe.

If these ideas are true, the outlook for our universe is hardly an optimistic one, even though vast ages will elapse before the end. But with true scientific candor, Sir James recognizes the limitations of our knowledge. "In some way," he says, "the material universe appears to be passing away like a tale that is told, dissolving into nothingness like a vision. The human race, whose intelligence dates back only a single tick of the astronomical clock, could hardly hope to understand so soon what it all means. Some day perhaps we shall know: at present we can only wonder."

Though the book is so fully up to date, it is indicative of the rapid progress of astronomy to note that one statement is already superseded. On page 94 he states that "very careful investigations" indicate the absence of absorbing matter in space, except for the dark nebulæ. Since these words were written, as recently as last summer, Dr. R. J. Trumpler, of the Lick Observatory, and later Dr. Piet Van de Kamp, of the McCormick Observatory, have shown, apparently very conclusively, that all through the galaxy there is a stratum of absorbing material. A slight error occurs on page 21, where the date of the first edition of Copernicus' "De Revolutionibus" is given as 1555 instead of 1543. Galileo is mentioned as the first and only discoverer of the satellites of Jupiter. This is a common error, though it now seems certain, as J. H. Johnson has shown recently in the *Journal of the British Astronomical Association*, that the Bavarian, Simon Marius, discovered them independently in the late fall of 1609, some weeks before Galileo. Also, teachers who try to make their students use *weight* and *mass* correctly will regret that Sir James constantly uses the former instead of the latter, even referring to the *weight* of the Earth.

But these are minor points. In the reviewer's opinion, this is the best popular introduction to astronomy now available. While there have been excellent texts, just as elementary, there was a real demand for a truly popular work. Now Sir James has produced it, and it is to be hoped that his book will enjoy a wide distribution. With astrology and similar pseudo-sciences now so widely disseminated, by newspaper and radio, there is a more urgent need than ever before for sound popular works such as this.

JAMES STOKLEY.

PROBABILITES ET STATISTIQUES. Par R. de Montessus de Ballore, Docteur es Sciences, Laureat de l'Institut; Preface de M. Alliaume, Professeur a l'Universite de Louvain. IX–211 pages, 8vo., paper, Paris, Librairie Scientifique Hermann et Cie., 1931. Price 60 francs.

Long in current use in such activities as the adjustment of statistics, actuarial processes, and, in science, in the adjustment of precise observations, the methods of probability are continually expanding over an ever-extending field. They have indeed become a well-nigh universal component in the mathematical expression of modern physical science. The author's development of the subject with much amplification and the derivation of directly applicable methods of statistical inference follows the modern transcription of that employed by James Bernoulli in his classic *Ars Conjectandi.* The work is a collection in didactic form of the author's researches which appeared in "Annales de la Societe Scientifique de Bruxelles" during the past seven years.

The basic theme of the study is the *function of simple probability*, as he terms it, otherwise known as the "binomial function":

$$y = \frac{m!}{(mp - x)! \, (mq + x)!} P^{mp-x} Q^{mq+x},$$

so called from the similarity of the factorial coefficient to that of a term in a binomial expansion. Here m is the number of trials, p the probability of occurrence in one trial, q the probability of failure and x the error. By means of the Stirling formula for the approximate equivalent of a factorial expression, is demonstrated how the binomial formula may assume the exponential form, familiar in the adjustment of observations. The analysis is searching and diversified and copiously illustrated by numerical examples of the application of the binomial formula to the most useful types of statistical data.

In the day of Bernoulli the acute interest of the present day in probability did not exist, nor were facilities for mathematical reduction like, for instance, Stirling's theorem available, but the present author enjoying these modern advantages provides convincing evidence of having employed them in making a real advance in sound methods of statistical inference.

L. E. P.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 366. Dynamic and Flight Tests on Rubber-Cord and Oleo-Rubber-Disk Landing Gears for an F6C–4 Airplane—by William C. Peck, 19 pages, illustrations, quarto, Washington, Government Printing Office, 1930, price ten cents.

This investigation was conducted at the request of the Bureau of Aeronautics, Navy Department, for the purpose of comparing an oleo-rubber-disk and a rubber-cord landing gear, built for use on an F6C–4 airplane. The investigation consisted of drop tests under various loading conditions and flight tests on an F6C–4 airplane. In the drop tests the total work done on each gear and the work done on each of the shock-absorbing units were determined. For both drop tests and flight tests the maximum loads and accelerations were determined.

The comparative results showed that the oleo gear was slightly superior in reducing the ordinary landing shocks, that it had a greater capacity for work, and that it was very superior in the reduction of the rebound. The results further showed that for drops comparable to very severe landings, the rubber-cord gear was potentially more effective as a shock-reducing mechanism. However, due to the construction of this chassis, which limited the maximum elongation of the cords, this gear was incapable of withstanding as severe tests as the oleo gear. The action of the oleo gear during the tests was greatly inferior to the action of an ideal gear. The maximum accelerations encountered during the flight tests for severe landings were 3.64g for the rubber-cord gear and 2.27g for the oleo gear. These were less than those experienced in free drops of 7 inches on either gear.

Report No. 367. Pressure Distribution over a Thick, Tapered and Twisted Monoplane Wing Model—N. A. C. A. 81-J—by Carl J. Wenzinger, 16 pages, illustrations, quarto, Washington, Government Printing Office, 1930, price ten cents.

This report presents the results of pressure distribution tests on a thick, tapered and twisted monoplane wing model. The investigation was conducted for the purpose of obtaining data on the aerodynamic characteristics of the new wing and to provide additional information suitable for use in the design of tapered cantilever wings. The tests included angles of attack up to 90 degrees and were made in the Atmospheric Wind Tunnel of the Committee.

The span loading over the wing was approximately of elliptical shape, which gave rise to relatively small bending moments about the root. The angle of zero lift for all sections along the span varied only within ± 0.4 degree of the angle of zero lift for the whole wing, resulting in small leading edge loads for the high-speed condition of flight. The results also add to the available information for the study of stability at large angles of attack.

Report No. 369. Maneuverability Investigation of the F6C-3 Airplane with Special Flight Instruments—by C. H. Dearborn and H. W. Kirschbaum, 21 pages, illustrations, quarto, Washington, Government Printing Office, 1930, price fifteen cents.

This investigation was made for the purpose of obtaining information on the maneuverability of the F6C-3 fighter airplane. The tests were conducted by the Committee at Langley Field, Va., at the request of the Bureau of Aeronautics, Navy Department. It is the first in a series of similar investigations to be conducted on a number of military airplanes for the purpose of comparing the abilities of these airplanes to maneuver, and also to establish a fund of quantitative data which may be used in formulating standards of comparison for rating the maneuverability of any airplane. A large part of this initial investigation was necessarily devoted to the development and trial of methods suitable for use in subsequent investigations of this nature.

Air speed, angular velocity, linear acceleration, and position of the control surfaces were measured by instruments in the airplane during loops, push-downs, pull-outs from dives, pull-ups from level flight, barrel rolls, and spins. The coördinates of the flight paths were deduced from the data whenever possible, and were checked in some cases by the use of a camera obscura. The results are given in curves showing the variation of the measured quantities with respect to time, and maximum values are tabulated.

Report No. 370. Effect of Variation of Chord and Span of Ailerons on Hinge Moments at Several Angles of Pitch—by B. H. Monish, 15 pages, illustrations, quarto, Washington, Government Printing Office, 1930, price ten cents.

This report presents the results of an investigation of the hinge moments of ailerons of various chords and spans on two airfoils having the Clark Y and U. S. A. 27 wing sections, supplementing the investigations described in References 1 and 2, of the rolling and yawing moments due to similar ailerons on these two airfoil sections.

The measurements were made at various angles of pitch, but at zero angle of roll and yaw, the wing chord being set at an angle of + 4° to the fuselage axis. In the case of the Clark Y airfoil the measurements have been extended to a pitch angle of 40°, using ailerons of span equal to 67 per cent. of the wing semispan and chord equal to 20 and 30 per cent. of the wing chord.

The work was done in the 10-foot tunnel of the Bureau of Standards on models of 60-inch span and 10-inch chord, having square tips, no taper in plan form or thickness, zero dihedral, and zero sweepback.

Report No. 372. The Gaseous Explosive Reaction—The Effect of Pressure on the Rate of Propagation of the Reaction Zone and Upon the Rate of Molecular Transformation—by F. W. Stevens, 19 pages, illustrations, quarto, Washington, Government Printing Office, 1930, price fifteen cents.

A study of the gaseous explosive reaction, the source of power in gas engines, has been in progress at the Bureau of Standards for a number of years as one of the projects undertaken at the request and with the support of the Committee. In a report on this work recently released, the effect of pressure on the rate of propagation of the explosive reaction zone and on the rate of energy liberation during the progress of the reaction is considered from a physico-chemical standpoint. The extensive experimental data on which the report is based, are drawn from the explosive reaction of a large number of fuels, covering reaction orders from three to fifteen. During their reaction these fuels were subjected to wide ranges of pressure and of other physical conditions usually imposed on the reaction by its industrial applications.

As a result of these studies, there have been brought out a number of important fundamental characteristics of the explosive reaction indicating that the basal processes of the transformation are much simpler and correspond more closely to the general laws and principles of ordinary transformations than is usually supposed.

Perhaps the most striking indication of the simplicity and uniformity of the reaction processes occurring in the explosive transformation is shown in those cases where it is so conditioned that it may run its course at a constant pressure. Under this condition the zone of explosive reaction, initiated at the point of ignition, assumes the form of a sphere, expanding, somewhat as a sound wave, within the explosive gases at a uniform rate. This constant linear rate at which the zone is observed to expand in space is not, however, the rate at which it propagates itself within the gases, thus effecting their transformation. This is because the gases themselves are also in movement outward from the point of ignition. The observed rate of displacement in space of the reaction zone has usually been mistaken for its rate of propagation relative to the gases. The report makes the necessary distinction and shows that the rate of propagation of the zone relative to the gases is independent of the mass movement of the gases. It is further shown that its rate of propagation relative to the gases is independent of the pressure to which they were subjected and that, as a consequence, the rate of molecular transformation within the zone of explosive reaction is proportional to pressure. The report also calls attention to the point that if these simple results are interpreted in the light of a statistical theory of impacts it would mean, since the rate of molecular transformation within the zone was found in all cases to be proportional to pressure, that the transformation within the zone is the result of binary impacts. This result is of unusual interest in the case of the reaction of heavy hydrocarbon fuels and the reaction mechanism proposed by the recent kinetic theory of Chain Reactions.

Report No. 373. Coefficients of Discharge of Fuel Injection Nozzles for Compression-Ignition Engines—by A. G. Gelalles, 19 pages, illustrations, quarto, Washington, Government Printing Office, 1931, price ten cents.

This report presents the results of an investigation to determine the coefficients of discharge of nozzles with small, round orifices of the sizes used with high speed compression-ignition engines. The injection pressures and chamber back pressures employed were comparable to those existing in compression-ignition engines during injection. The construction of the nozzles was varied to determine the effect of the nozzle design on the coefficient. Tests were also made with the nozzles assembled in an automatic injection valve, both with a plain and with a helically grooved stem.

It was found that a smooth passage before the orifice is requisite for high flow efficiency. A beveled leading edge before the orifice gave a higher coefficient of discharge than a rounded edge. Varying the length-diameter ratio from 1 to 3 for one of the orifices having a beveled leading edge was found to have no effect on the value of the coefficient. The results with the nozzles assembled in an automatic injection valve having a plain stem duplicated those with the nozzles assembled at the end of a straight tube of constant diameter. Lower coefficients were obtained with the nozzles assembled in an injection valve having a helically grooved stem.

When the coefficients of nozzles of any one geometrical shape were plotted against values of corresponding Reynolds Numbers for the orifice diameters and rates of flow tested, it was found that experimental points were distributed along a single curve.

Report No. 376. Some Approximate Equations for the Standard Atmosphere—by Walter S. Diehl, 12 pages, illustrations, quarto, Washington, Government Printing Office, 1931, price ten cents.

This report, which was prepared for publication by the Committee, contains the derivation of a series of simple approximate equations for density ratios ρ/ρ_0, ρ_0/ρ, $\sqrt{\rho_0/\rho}$ and for the pressure ratio ρ/ρ_0, in the standard atmosphere. The accuracy of the various equations is discussed and the limits of applications are given. Several of these equations are in excellent agreement with the standard values.

R.

AN INTRODUCTION TO STRUCTURAL THEORY AND DESIGN THEORY. By Hale Sutherland, A.B., S.B., M. Am. Soc. C. E., Professor of Civil Engineering, Lehigh University, and Harry Lake Bowman, S.M., M. Am. Soc. C. E., Professor of Civil Engineering, Drexel Institute. ix–318 pages, 8vo, cloth. New York, John Wiley & Sons, Inc., 1930. Price $3.50.

A steady increase of good text-books on structural mechanics has followed the enormous expansion of industrial activity of the last few decades. Principles necessarily remain unchanged and duplication is inevitable, but each successive book affords an opportunity to introduce better methods of procedure, greater pedagogical clarity, and occasionally a more extensive consideration of theorems which in the past have not received as much attention as the searching analysis essential in the design of the very large structures of the present day demands. The authors have not been remiss in contributing their quota to that sort of betterment in text-book making, and so sharing in providing better facilities in the now major industry of "teaching the young idea."

In general, the work deals with the conventional subject of roofs and bridges, tall building frames and space framework, that is, structures whose members cannot be treated as lying in one plane. The graphic method is considerably stressed throughout the work, and a generous chapter on graphic statics adequately prepares the student for its application to the analysis of trusses. The influence line, now much in favor for studying the effects of concentrated loads, receives an extensive application that will not fail to make its meaning and value clear to the student. For erection purposes and the determination of the stresses in statically indeterminate structures, the calculation of slope and deflection are essential. An ample and, for a text-book, a distinctive chapter is devoted to this important topic. Therein are described at length the method of work, the method of elastic weights and moment-area theorems, the bar-chain method, and the Williot diagram for frame deformation. The differential expressions M/EI dx and M/EI xdx of the method of work, and the moment-area theorems will be recognized as a rarely useful device now widely employed in practice and adopted by recent writers on the mechanics of materials. It may be noted that these differential expressions also represent elements of the moment and moment area or statical moment respectively. It is not easy to agree with the authors' statement that the moment-area method is cumbersome when applied to other than simple beams: our own experience has been indeed quite the opposite.

Rigid frames and continuity are analyzed in accordance with the methods previously discussed. Here the authors introduce a recent (1929) mode of analysis of rigid frames, developed by Professor Hardy Cross of the University of Illinois, known as the method of moment distribution, which is designed to simplify the determination of rigid frame structures, particularly those of reinforced concrete. Wind stresses in tall buildings have a well-illustrated chapter of current methods of dealing with that element in building construction, and a note on earthquake stresses. On indeterminate trusses, the methods of least work, of deflections, and of influence lines are developed. Secondary stresses and space frame-work have each an ample chapter. A copious appendix contains specifications of loads on structures with numerous tables, charts and diagrams.

The book is obviously planned with care. Illustrative examples, fully worked out invariably accompany the text with an adequate number of problems for the student to solve himself. As a school book, it is comprehensive in character and a bit formidable here and there: but the authors are generous with explanatory matter, and the serious student will encounter few risks of becoming hopelessly entangled in a morass of difficult deduction. As a reference work it should prove a welcome addition to the library of the practicing engineer.

L. E. P.

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CURRENT TOPICS.

The Metal Crystal. SIR HAROLD SARPENTER. (*Nature*, July 5, 1930.) It would be difficult, if not impossible, to discover elsewhere in the compass of a dozen pages a statement of present knowledge on the subject of metal crystals as authoritative and informing as this report of a discourse at the Royal Institution delivered May 30, 1930.

"When prepared by one of the usual methods, metals and alloys consist of an aggregate of small allotriomorphic crystals. Pure metals usually consist of a large number of similar crystals. Alloys are sometimes composed of one type of crystal, though usually they contain two or more different kinds. In both cases metals as ordinarily prepared contain anything from about one hundred thousand to several millions of crystals per cubic inch. In all metals, however, the individual crystal is the unit of which the aggregate is built. It is therefore the simplest form of metal." If the properties of such an individual crystal were different in different directions this could not be discovered by a study of an aggregate of crystals, because the constituent crystals have all possible orientations. Furthermore the properties of the crystal boundaries have great influence in determining the properties of the aggregate. "It has long been known that the boundaries in a crystal aggregate are stronger than the interiors of the crystals. For example, a metal fractures under tensile test through and not around the crystals. One of the commonest methods of increasing the strength of a metal or alloy is to diminish its crystal size and increase its boundary area by suitable thermal and mechanical treatment." A study of the properties of a single crystal thus gives information of a fundamental nature, free of the complications due to the boundaries and various orientations of other crystals. About a dozen years ago investigators began to prepare single crystals of metals for examination. Success has been attained in three ways:

(1) By production of the crystal from the vapor phase.
(2) By its production from the liquid phase.
(3) By the conversion of the solid metal in the ordinary polycrystalline aggregate into a single crystal.

Today there are nine methods of producing single metal crystals—two from the vapor, three from the liquid, and four from the solid phase.

Both methods for obtaining single crystals from vapors grew out of the technique devised in the manufacture of incandescent lamps for the deposition of tungsten on glowing filaments. The following is the process used by de Boer and Fast for growing zirconium single crystals. A glowing filament of tungsten was surrounded by zirconium iodide in the state of vapor. The vapor is decomposed by the heat and zirconium is deposited on the filament. The careful maintenance of the temperature of the filament at about 1800° C. is requisite for the success of the method. Too low a temperature results in polycrystalline zirconium, while too high a temperature produces a eutectic of tungsten and zirconium. In a successful operation the zirconium rod deposited around the tungsten filament is built up of single zirconium crystals occupying the entire cross-section of the rod and varying in length from .5 cm. to 1.5 cm. The zirconium prepared as above described, whether mono- or polycrystalline, is ductile, whereas as prepared previously it was brittle by reason of the presence of traces of impurities. By the same method single crystals of titanium, hafnium and thorium have been obtained. Koref by a different process has formed single crystals of molybdenum, tantalum, iron, zirconium and titanium. Deposition from the vapor phase is especially applicable to the production of single crystals of metals with high melting points, provided the iodides of the metals satisfy a certain condition.

One way of getting single crystals from liquids is this. Molten metal in a container made of refractory material is lowered vertically through an electric furnace that keeps the liquid just above its melting point. The lower end, tapered to a point, is the first to leave the furnace and it is there that solidification starts. As the lowering of the metal continues solidification extends progressively to layers of liquid above the point. With a suitable velocity single crystals are commonly formed By this means Bridgman prepared single crystals of tin, zinc, cadmium, antimony, bismuth and tellurium. For bars 2.2 cm. in diameter the speed of lowering was about 4 mm. per hour. For thinner crystals a higher speed was suitable. Another method depends on drawing the molten metal up above the liquid surface by an auxiliary wire. These two methods have been mainly used with metals of low melting points.

Conversion of a solid crystal aggregate into one or more single crystals depends upon straining the aggregate and afterwards heating it. In the case of aluminium the metal was first softened by heating for six hours at 550° C. A tensile stress of 2.4 tons per sq. in. was next applied. While still under strain the metal was heated to 450° C. and the temperature was raised about 25° a day until it reached 550°. Then for one hour it was kept at 600°. By this process single crystal bars of aluminium have been produced about 8 in. long and .564 in. in diameter. For obtaining a single crystal the strain must be exactly right. "In this way, upwards of seven million of the original crystals can be caused to coalesce into a single crystal." This method is more general than those in which single crystals are obtained from vapors or liquids, because, by the adjustment of temperature and strain, crystals of specified size may be produced. Furthermore, it can be used with metals without regard to the location of their melting points.

"The ease with which single crystal bars of aluminium, copper, silver and gold, and even iron, can be deformed is astonishing. Each of these metals can be bent very readily, but once it has been bent to any marked extent, a higher stress is required to make it revert to its original shape." It has been found that aluminium under tension has no limit of elasticity when the observation is made on a single crystal, but that it is deformed plastically no matter how small the applied stress. Hence it appears that the possession of an elastic limit by ordinary aluminium is dependent upon the presence of a multitude of crystals.

G. F. S.

Coagulation Conditions and Bacterial Efficiency of Preliminary Water Treatment. The United States Public Health Service has recently announced the results of certain experimental studies relating to coagulation conditions and bacterial efficiency of preliminary water treatment. The studies recorded in this report are concerned with the effects of (*a*) the period of sedimentation, (*b*) the method of applying coagulant to the raw water, (*c*) the hydrogen ion concentration of the coagulation reaction, and (*d*) the density of coagulant added to the raw water, on the bacterial efficiency of coagulation-sedimentation as a preparatory treatment of water for rapid sand filtration and, incidentally, on the efficiency of the entire process of rapid sand filtration.

The experiments were carried out during portions of the three years, 1926, 1927, and 1928, at a fully equipped experimental rapid sand water filtration plant of 160,000 gallons daily capacity, designed to typify, as nearly as possible, current large-scale practice in this process of water purification, but with certain additional features incorporated in the plant for experimental purposes.

The results of the study yielded the following main conclusions:

(1) Substantial gains in the bacterial efficiency of coagulation-sedimentation resulted from prolongation of the nominal sedimentation period up to 8 or 9 hours, and measurable gains with periods up to 12 hours.

(2) Variations in the hydrogen ion concentration in the coagulation reaction from 5.6 to 6.9 produced little effect on the efficiency of coagulation-sedimentation. The efficiency became sharply diminished, however, with hydrogen ion concentration values exceeding 7.0 and slightly improved with hydrogen ion concentration values approaching 5.5.

(3) The bacterial efficiency of double-stage coagulation, with two separate stages of sedimentation, was consistently greater than that of single-stage coagulation with one stage of sedimentation. The observations indicated, however, that with the same total amount of coagulant and the same total period of sedimentation, little if any difference was observable between the results shown by double-stage and single-stage coagulation when carried out in conjunction with two separate states of sedimentation.

(4) A fairly consistent relation was shown between the amounts of coagulant added to the raw water and the resulting bacterial efficiency, both of coagulation-sedimentation and of this state in conjunction with filtration. This relationship was found to hold irrespective of raw-water turbidity or bacterial content, though it was more apparent when the turbidity and bacterial numbers were higher. Measurable gains in efficiency were shown with increases in coagulant density ranging up to 5 grains per gallon.

The general conclusion reached from the foregoing series of experiments was that the bacterial efficiency of rapid sand filtration processes can be increased very materially by means of longer periods of sedimentation and larger amounts of coagulant than ordinarily are used in current water purification practice. The economical limit of sedimentation appears to be reached somewhere between 8 and 12 hours, with little gain in efficiency beyond the upper limit of time stated.

R.

The Loss of Light Due to Smoke in New York City—U. S. Public Health Service, Health News. The contamination of the atmosphere by smoke from the chimneys of private houses, office buildings, industrial plants, steam engines, tugs, and steamships has become a serious matter in several of the larger cities of the United States. The presence in the air of large numbers of particles of soot and ash, and of appreciable amounts of sulphuric acid and other impurities results in injury to trees and plants and in economic loss, and is a detriment to health.

Since the injury to health is due, in part, to the loss of daylight, and since the loss of daylight is a measure of the smokiness of the atmosphere, it seems desirable to determine, as far as possible, the exact amount of light which is lost because of smoke in a large city in different seasons of the year and at different hours of the day; and also the extent to which the loss of light is affected by the height, shape, and density of the smoke layer, the humidity of the air, the velocity and direction of the wind, and by other factors.

A favorable opportunity for such a study offered itself in New York City during the year 1927, since simultaneous records of daylight could be conveniently made on the roof of the United States Marine Hospital at 67 Hudson Street, at the lower end of Manhattan Island, where the air was unusually smoky, and on the roof of a building at the United States Quarantine Station on Hoffman Island in lower New York Bay, about 9 miles south of the hospital, where the air was comparatively clear. The instruments used were of the type which have been in use in Washington, D. C., by the United States Public Health Service for recording daylight since July, 1924. They consisted of photoelectric cells and recording potentiometers.

The photoelectric cell was placed on the roof of the building where the study was to be made and the recording potentiometer was placed in a convenient position within the building. The platform was of such height that no shadows were cast upon the cell at any season of the year, and was also so constructed that the cell was accessible, so that its glass surface could be kept clean. The cell at Hudson Street was about 79 feet above the street level and 93 feet above mean high water. The cell on Hoffman Island was about 35 feet above the ground and 40 feet above high water.

Comparison of the results obtained at the two places showed that there was a large relative loss of light at the Hudson Street Hospital due to smoke. In some cases the average hourly or daily loss was greater than 50 per cent. The average percentage loss for the whole year was 16.6 for clear days, 34.6 for cloudy days, and 21.5 for all days. The results showed that the loss of light depends, among other things, upon the altitude of the sun, upon the nature of the daylight, whether from a clear or a cloudy sky, upon the relative humidity of the air, and upon the velocity of the wind.

The effect of the altitude of the sun upon the percentage loss of light was shown clearly in the variation of the percentage loss with the hour of the day. The average percentage loss throughout the year was 30.2 at 8:30 A.M., 16.5 at 1:30 P.M., and 21.0 at 3:30 P.M. The average monthly percentage losses of light showed no marked seasonal effect but did show a marked relation to the average monthly relative humidities, the percentage losses usually increasing and decreasing with the relative humidities. For clear days the greatest average monthly percentage loss was 23.1 in November and the least 12.1 in May, with corresponding average relative humidities of 64.6 and 39.0 per cent. For cloudy days these values were 52.7 in September and 24.6 in December, with corresponding average relative humidities of 99.4 and 85.9 per cent.

Other conditions being the same, the average percentage loss of light was greater for cloudy days, or cloudy hours, than for clear days, or clear hours; the percentage loss being about 1.5 times as great for cloudy days as for clear days for relative humidities between 40 and 80 per cent. and for wind velocities between 10.0 and 19.9 miles per hour. For the same kind of sky, clear or cloudy, the average percentage loss of light increased with increase of relative humidity. For a clear sky the average percentage loss was twice as great for a relative humidity of 65 per cent. as for 35 per cent. For a cloudy sky the increase with relative humidity was not as great. The percentage loss of light was also found to decrease as the velocity of the wind increased.

R.

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